



Exercise set (0.2)

Exercise set 0.2:

27, 28, 29 (a, b, c, f), 30 (b, c, g), 31, 32, 34, 35, 41, 42, 43, 44, 59 (a, b, e, f) p.25-26.

27–28 Find formulas for $f + g$, $f - g$, fg , and f/g , and state the domains of the functions. ■

27. $f(x) = 2\sqrt{x-1}$, $g(x) = \sqrt{x-1}$

$$(f+g)(x) = 2\sqrt{x-1} + \sqrt{x-1} = 3\sqrt{x-1}$$

Domain:

$$x-1 \geq 0$$

$$x \geq 1$$

$$D_{f+g} = [1, +\infty)$$

$$(f-g)(x) = 2\sqrt{x-1} - \sqrt{x-1} = \sqrt{x-1}$$

Domain:

$$x-1 \geq 0$$

$$x \geq 1$$

$$D_{f-g} = [1, +\infty)$$

$$(fg)(x) = (2\sqrt{x-1})(\sqrt{x-1}) = 2(x-1) = 2x-2$$

Domain:

$$D_{fg} = [1, +\infty)$$

$$(f/g)(x) = \frac{2\sqrt{x-1}}{\sqrt{x-1}} = 2$$

Domain:

$$x-1 > 0$$

$$x > 1$$

$$D_{f/g} = [1, +\infty) \cap (1, +\infty) = (1, +\infty)$$

28. $f(x) = \frac{x}{1+x^2}$, $g(x) = \frac{1}{x}$

$$\begin{aligned}(f+g)(x) &= \frac{x}{1+x^2} + \frac{1}{x} \\ &= \frac{x^2 + (1+x^2)}{x(1+x^2)} = \frac{2x^2 + 1}{x(1+x^2)}\end{aligned}$$

Domain:

$$D_f = (-\infty, +\infty)$$

$$D_g = (-\infty, 0) \cup (0, +\infty)$$

$$\begin{aligned}D_{f+g} &= (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty) \\ &= (-\infty, 0) \cup (0, +\infty)\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= \frac{x}{1+x^2} - \frac{1}{x} \\ &= \frac{x^2 - (1+x^2)}{x(1+x^2)} = \frac{-1}{x(1+x^2)}\end{aligned}$$

Domain:

$$D_f = (-\infty, +\infty)$$

$$D_g = (-\infty, 0) \cup (0, +\infty)$$

$$\begin{aligned}D_{f-g} &= (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty) \\ &= (-\infty, 0) \cup (0, +\infty)\end{aligned}$$

$$-(fg)(x) = \frac{x}{1+x^2} \cdot \frac{1}{x} = \frac{1}{1+x^2}$$

Domain:

$$D_f = (-\infty, +\infty)$$

$$D_g = (-\infty, 0) \cup (0, +\infty)$$

$$\begin{aligned} D_{f \cdot g} &= (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty) \\ &= (-\infty, 0) \cup (0, +\infty) \end{aligned}$$

$$\begin{aligned} -(f/g)(x) &= \frac{x}{1+x^2} / \frac{1}{x} \\ &= x \cdot \frac{x}{1+x^2} = \frac{x^2}{1+x^2} \end{aligned}$$

Domain:

$$D_f = (-\infty, +\infty)$$

$$D_g = (-\infty, 0) \cup (0, +\infty)$$

$$\begin{aligned} D_{f/g} &= (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty) \\ &= (-\infty, 0) \cup (0, +\infty) \end{aligned}$$

29. Let $f(x) = \sqrt{x}$ and $g(x) = x^3 + 1$. Find

(a) $f(g(2))$

$$f(g(x)) = f(x^3 + 1) = \sqrt{x^3 + 1}$$

$$f(g(2)) = \sqrt{2^3 + 1} = \sqrt{8 + 1} = \sqrt{9} = 3$$

(b) $g(f(4))$

$$g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^3 + 1$$

$$g(f(4)) = (\sqrt{4})^3 + 1 = 8 + 1 = 9$$

(c) $f(f(16))$

$$f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}}$$

$$f(f(16)) = \sqrt{\sqrt{16}} = 2$$

(f) $g(3 + h)$.

$$g(3+h) = (3+h)^3 + 1$$

30. Let $g(x) = \sqrt{x}$. Find

(b) $g(\sqrt{x} + 2)$

$$g(\sqrt{x} + 2) = \sqrt{\sqrt{x} + 2}$$

(c) $3g(5x)$

$$3g(5x) = 3\sqrt{5x}$$

(g) $g(1/\sqrt{x})$

$$g(1/\sqrt{x}) = \sqrt{\frac{1}{\sqrt{x}}} = \frac{1}{\sqrt{\sqrt{x}}} = \frac{1}{\sqrt[4]{x}}$$

31–34 Find formulas for $f \circ g$ and $g \circ f$, and state the domains of the compositions. ■

31. $f(x) = x^2$, $g(x) = \sqrt{1-x}$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(\sqrt{1-x}) \\ &= (\sqrt{1-x})^2 \\ &= 1-x\end{aligned}$$

$$D_{f \circ g} = D_g \cap D_R$$

$$D_g = 1-x \geq 0 \longrightarrow x \leq 1 \longrightarrow (-\infty, 1]$$

$$D_R = (-\infty, +\infty)$$

$$D_{f \circ g} = (-\infty, 1] \cap (-\infty, +\infty) = (-\infty, 1]$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(x^2) \\ &= \sqrt{1-x^2}\end{aligned}$$

$$D_{g \circ f} = D_f \cap D_R$$

$$D_f = (-\infty, +\infty)$$

$$D_R = 1-x^2 \geq 0 \longrightarrow 1 \geq x^2 \longrightarrow \sqrt{1} \geq \sqrt{x^2}$$

$$|x| \leq 1 = -1 \leq x \leq 1 = [-1, 1]$$

$$D_{g \circ f} = (-\infty, +\infty) \cap [-1, 1] = [-1, 1]$$

$$32. f(x) = \sqrt{x-3}, \quad g(x) = \sqrt{x^2+3}$$

$$-(f \circ g)(x) = f(g(x)) = f(\sqrt{x^2+3}) = \sqrt{\sqrt{x^2+3}-3}$$

$$-D_{f \circ g} = D_g \cap D_R$$

$$D_g = (-\infty, +\infty)$$

$$D_R = \sqrt{x^2+3} - 3 \geq 0 \rightarrow \sqrt{x^2+3} \geq 3$$

$$(\sqrt{x^2+3})^2 \geq (3)^2 \rightarrow x^2+3 \geq 9$$

$$x^2 \geq 6 \rightarrow \sqrt{x^2} \geq \sqrt{6}$$

$$|x| \geq \sqrt{6} \rightarrow x \geq \sqrt{6} \text{ or } x \leq -\sqrt{6}$$

$$D_R = (-\infty, -\sqrt{6}] \cup [\sqrt{6}, +\infty)$$

$$D_{f \circ g} = (-\infty, +\infty) \cap (-\infty, -\sqrt{6}] \cup [\sqrt{6}, +\infty)$$

$$= (-\infty, -\sqrt{6}] \cup [\sqrt{6}, +\infty)$$

$$-(g \circ f)(x) = g(f(x)) = g(\sqrt{x-3}) = \sqrt{\sqrt{(x-3)^2}-3}$$

$$= \sqrt{x-3+3} = \sqrt{x}$$

$$-D_{g \circ f} = D_f \cap D_R$$

$$D_f = X - 3 \geq 0 \xrightarrow{\text{red}} X \geq 3 = [3, +\infty)$$

$$D_R = X \geq 0 = [0, +\infty)$$

$$D_{g \circ f} = [3, +\infty) \cap [0, +\infty) = [3, +\infty)$$

$$34. f(x) = \frac{x}{1+x^2}, \quad g(x) = \frac{1}{x}$$

$$-(f \circ g)(x) = f(g(x)) = f(1/x) = \frac{1/x}{1 + (1/x)^2}$$

$$= \frac{1/x}{1 + 1/x^2} = \frac{1/x}{\frac{x^2 + 1}{x^2}}$$

$$= \frac{x^2}{x^2 + 1} \cdot \frac{1}{x}$$

$$-D_{f \circ g} = D_g \cap D_R$$

$$D_g = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

$$D_R = \mathbb{R} = (-\infty, +\infty)$$

$$D_{f \circ g} = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

$$-(g \circ f)(x) = g(f(x)) = g\left(\frac{x}{1+x^2}\right)$$

$$= \frac{1}{\frac{x}{1+x^2}} = \frac{1+x^2}{x}$$

$$-D_{g \circ f} = D_f \cap D_R$$

$$D_f = \mathbb{R} = (-\infty, +\infty)$$

$$D_R = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

$$D_{g \circ f} = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

35–40 Express f as a composition of two functions; that is, find g and h such that $f = g \circ h$. [Note: Each exercise has more than one solution.] ■

35. (a) $f(x) = \sqrt{x+2}$

$$g(x) = \sqrt{x}$$

$$h(x) = x+2$$

$$g \circ h = g(h(x)) = g(x+2) = \sqrt{x+2}$$

(b) $f(x) = |x^2 - 3x + 5|$

$$g(x) = |x|$$

$$h(x) = x^2 - 3x + 5$$

$$g \circ h = g(h(x)) = g(x^2 - 3x + 5) = |x^2 - 3x + 5|$$

41–44 True–False Determine whether the statement is true or false. Explain your answer. ■

41. The domain of $f + g$ is the intersection of the domains of f and g . (T)

42. The domain of $f \circ g$ consists of all values of x in the domain of g for which $g(x) \neq 0$. (F)

Domain of $f \circ g$ Consist of all x in the domain of g
for which $g(x)$ in the domain of f

43. The graph of an even function is symmetric about the y -axis. (T)

44. The graph of $y = f(x + 2) + 3$ is obtained by translating the graph of $y = f(x)$ right 2 units and up 3 units. (F)

by translating the graph left 2 units and
up 3 units

59. In each part, classify the function as even, odd, or neither.

(a) $f(x) = x^2$

$$f(-x) = (-x)^2 = x^2 = f(x) \rightarrow \text{even function}$$

(b) $f(x) = x^3$

$$f(-x) = (-x)^3 = -x^3 \neq f(x) \rightarrow \text{not even}$$

$$f(-x) = -f(x) \rightarrow \text{odd function}$$

(c) $f(x) = \frac{x^5 - x}{1 + x^2}$

$$f(-x) = \frac{(-x)^5 - (-x)}{1 + (-x)^2} = \frac{-x^5 + x}{1 + x^2}$$

$$= -\frac{x^5 - x}{1 + x^2}$$

$$= -f(x) \rightarrow \text{odd function}$$

(d) $f(x) = 2$

$$f(-x) = 2 = f(x) \rightarrow \text{even function}$$