



Section (4.3) :

► **Example 1** Evaluate $\int (x^2 + 1)^{50} \cdot 2x dx$.

$$\int (x^2 + 1)^{50} 2x dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\int u^{50} du$$

$$= \frac{u^{50+1}}{50+1} + C$$

$$= \frac{u^{51}}{51} + C$$

$$= \frac{(x^2 + 1)^{51}}{51} + C$$

Guidelines for u -Substitution

Step 1. Look for some composition $f(g(x))$ within the integrand for which the substitution

$$u = g(x), \quad du = g'(x) dx$$

produces an integral that is expressed entirely in terms of u and its differential du . This may or may not be possible.

Step 2. If you are successful in Step 1, then try to evaluate the resulting integral in terms of u . Again, this may or may not be possible.

Step 3. If you are successful in Step 2, then replace u by $g(x)$ to express your final answer in terms of x .

■ EASY TO RECOGNIZE SUBSTITUTIONS

The easiest substitutions occur when the integrand is the derivative of a known function, except for a constant added to or subtracted from the independent variable.

► Example 2

$$\int \sin(x + 9) dx =$$

$$u = x + 9$$

$$du = dx$$

$$\int \sin(u) du$$

$$= -\cos(u) + C$$

$$= -\cos(x + 9) + C$$

$$\int (x - 8)^{23} dx =$$

$$u = x - 8$$

$$du = dx$$

$$\int u^{23} du$$

$$= \frac{u^{24}}{24} + C$$

$$= \frac{(x - 8)^{24}}{24} + C$$

► **Example 3** Evaluate $\int \cos 5x dx$.

$$u = 5x$$

$$du = 5 dx \rightarrow \frac{du}{5} = dx$$

$$\int \cos(u) \frac{du}{5}$$

$$= \frac{1}{5} \int \cos(u) du$$

$$= \frac{1}{5} \sin(u) + C$$

$$= \frac{1}{5} \sin(5x) + C$$

► Example 4

$$\int \frac{dx}{\left(\frac{1}{3}x - 8\right)^5} =$$

$$u = \frac{1}{3}x - 8$$

$$du = \frac{1}{3} dx \rightarrow 3 du = dx$$

$$\int \frac{3}{u^5} du$$

$$3 \int u^{-5} du$$

$$= 3 \frac{u^{-5+1}}{-5+1} + C$$

$$= 3 \frac{u^{-4}}{-4} + C$$

$$= -\frac{3}{4} u^{-4} + C$$

$$= -\frac{3}{4} \cdot \frac{1}{u^4} + C$$

$$= -\frac{3}{4} \cdot \frac{1}{\left(\frac{1}{3}x - 8\right)^4} + C$$

► **Example 5**

$$\int \left(\frac{1}{x} + \sec^2 \pi x \right) dx =$$

$$\int \frac{1}{x} dx + \int \sec^2(\pi x) dx$$

$$= \ln|x| + \int \sec^2(\pi x) dx$$

$$\int \sec^2(\pi x) dx$$

$$u = \pi x \quad dx$$

$$du = \pi dx \rightarrow \frac{du}{\pi} = dx$$

$$\int \sec^2(u) \frac{du}{\pi}$$

$$= \frac{1}{\pi} \int \sec^2(u) du$$

$$= \frac{1}{\pi} \tan(u) + C$$

$$= \frac{1}{\pi} \tan(\pi x) + C$$

$$\int \frac{1}{x} dx + \int \sec^2(\pi x) dx$$

$$= \ln|x| + \frac{1}{\pi} \tan(\pi x) + C$$

► **Example 6** Evaluate $\int \sin^2 x \cos x dx$.

$$u = \sin x$$

$$du = \cos x dx$$

$$\int u^2 du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{\sin^3 x}{3} + C$$

► **Example 7** Evaluate $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$.

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$2\sqrt{x} du = dx$$

$$\int \frac{\cos u}{\sqrt{x}} \cdot 2\sqrt{x} du$$

$$2 \int \cos u du$$

$$= 2 \sin u + C$$

$$= 2 \sin \sqrt{x} + C$$

► **Example 8** Evaluate $\int t^4 \sqrt[3]{3-5t^5} dt$.

$$u = 3 - 5t^5$$

$$du = -25t^4 dt$$

$$-\frac{du}{25} = t^4 dt$$

$$\int \sqrt[3]{u} \cdot -\frac{du}{25}$$

$$= -\frac{1}{25} \int u^{1/3}$$

$$= -\frac{1}{25} \cdot \frac{u^{4/3}}{4/3} + C$$

$$= -\frac{1}{25} \cdot \frac{3}{4} u^{4/3} + C$$

$$= -\frac{3}{100} (3 - 5t^5)^{4/3}$$

LESS APPARENT SUBSTITUTIONS

The method of substitution is relatively straightforward, provided the integrand contains an easily recognized composition $f(g(x))$ and the remainder of the integrand is a constant multiple of $g'(x)$. If this is not the case, the method may still apply but may require more computation.

► **Example 9** Evaluate $\int x^2 \sqrt{x-1} dx$.

$$u = x - 1$$

$$du = dx$$

$$u = x - 1 \rightarrow u + 1 = x \rightarrow (u + 1)^2 = x^2$$

$$(u^2 + 2u + 1) = x^2$$

$$\int (u^2 + 2u + 1) \sqrt{u} du$$

$$\int (u^2 + 2u + 1) u^{1/2} du$$

$$\int u^{2+1/2} + 2u^{1+1/2} + u^{1/2} du$$

$$\int u^{5/2} du + 2 \int u^{3/2} du + \int u^{1/2} du$$

$$\frac{u^{5/2+1}}{5/2+1} + 2 \frac{u^{3/2+1}}{3/2+1} + \frac{u^{1/2+1}}{1/2+1} + C$$

$$\frac{u^{7/2}}{7/2} + 2 \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C$$

$$\frac{2}{7} u^{7/2} + 2 \cdot \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C$$

$$\frac{2}{7} (x-1)^{7/2} + \frac{4}{5} (x-1)^{5/2} + \frac{2}{3} (x-1)^{3/2} + C$$

► **Example 10** Evaluate $\int \cos^3 x \, dx$.

$$\int \cos^2 x \cos x \, dx$$

$$\sin^2 x + \cos^2 x = 1$$

$$\int (1 - \sin^2 x) \cos x \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$\int (1 - u^2) \, du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$