0.2 NEW FUNCTIONS FROM OLD

العمليات

ARITHMETIC OPERATIONS ON FUNCTIONS

0.2.1 DEFINITION Given functions f and g, we define

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$(f/g)(x) = f(x)/g(x)$$

نعرف تقاطع

For the functions $f+g$, $f-g$, and fg we define the domain to be the intersection of the domains of f and g, and for the function f/g we define the domain to be the intersection of the domains of f and g but with the points where $g(x) = 0$ excluded (to avoid division by zero). باستثناء

IQ ACADEMY

▶ Example 1 Let

$$f(x) = 1 + \sqrt{x-2} \quad \text{and} \quad g(x) = x - 3$$

Find the domains and formulas for the functions $f+g$, $f-g$, fg, f/g, and $7f$.

Solution.

▶ **Example 2** Show that if $f(x) = \sqrt{x}$, $g(x) = \sqrt{x}$, and $h(x) = x$, then the domain of fg is not the same as the natural domain of h.

COMPOSITION OF FUNCTIONS

تركيب

IQ ACADEMY

0.2.2 DEFINITION Given functions f and g, the ***composition*** of f with g, denoted by $f \circ g$, is the function defined by

$$(f \circ g)(x) = f(g(x))$$

تحتوي

The domain of $f \circ g$ is defined to consist of all x in the domain of g for which $g(x)$ is in the domain of f.

▶ Example 3 Let $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$. Find

(a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$

Solution (a)

Solution (b)

► **Example 4** Find $(f \circ g \circ h)(x)$ if

$$f(x) = \sqrt{x}, \quad g(x) = 1/x, \quad h(x) = x^3$$

التعبير

EXPRESSING A FUNCTION AS A COMPOSITION

▶ **Example 5** Express $\sin(x^3)$ as a composition of two functions.

IQ ACADEMY

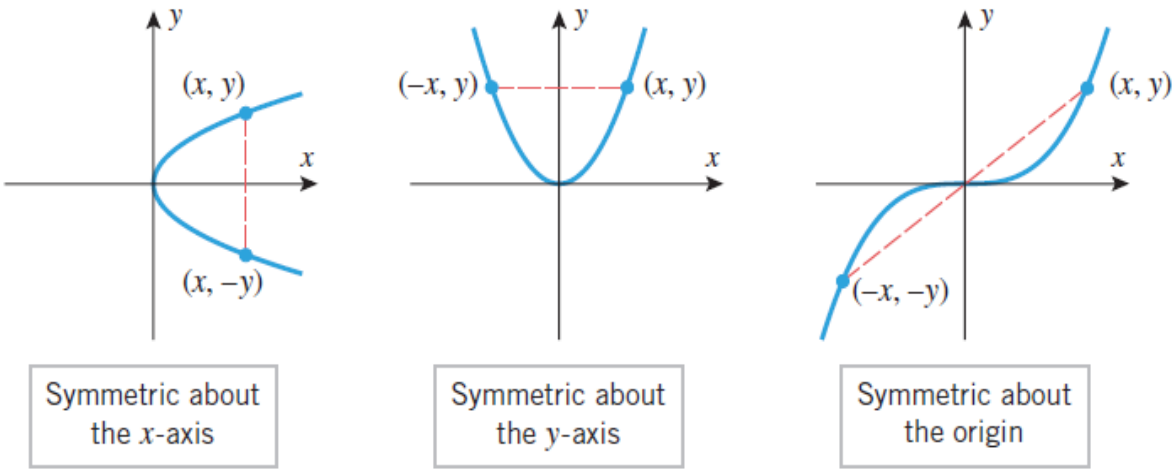
Solution

FUNCTION	*g(x)* *INSIDE* داخلية	*f(x)* *OUTSIDE* خارجية	*COMPOSITION*

REMARK

SYMMETRY

التماثل



0.2.3 THEOREM (*Symmetry Tests*)

تبديل المنحنى

(*a*) *A plane curve is symmetric about the y-axis if and only if replacing x by −x in its equation produces an equivalent equation.*

تنتج مكافئة معادلة

(*b*) *A plane curve is symmetric about the x-axis if and only if replacing y by −y in its equation produces an equivalent equation.*

(*c*) *A plane curve is symmetric about the origin if and only if replacing both x by −x and y by −y in its equation produces an equivalent equation.*

الزوجية الفردية

EVEN AND ODD FUNCTIONS

A function f is said to be an ***even function*** if

$$f(-x) = f(x) \tag{8}$$

and is said to be an ***odd function*** if

$$f(-x) = -f(x) \tag{9}$$

Geometrically, the graphs of even functions are symmetric about the y-axis

تبديل

because replacing x by $-x$ in the equation $y = f(x)$ yields $y = f(-x)$, which is

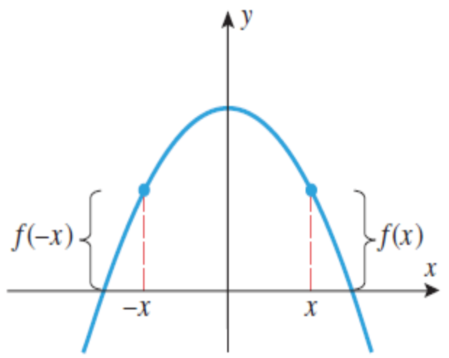
تساوي الأصلية المعادلة

equivalent to the original equation $y = f(x)$ by (8) (see Figure 0.2.9).

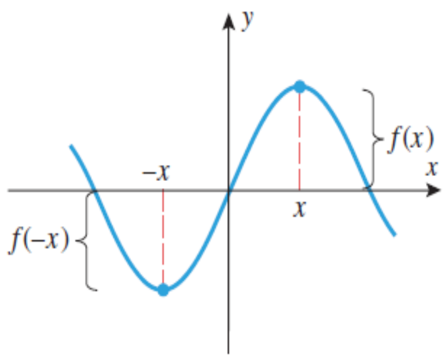
Similarly, it follows from (9) that graphs of odd functions are symmetric about the

نقطة الأصل

origin (see Figure 0.2.10).



▲ **Figure 0.2.9** This is the graph of an even function since $f(-x) = f(x)$.



▲ **Figure 0.2.10** This is the graph of an odd function since $f(-x) = -f(x)$.

Some examples of even functions:

Some examples of odd functions:

▸ **Example**

صنف

Exercise 59 Page 26: In each part, classify the function as even, odd, or neither

(c) $f(x) = |x|$

(e) $f(x) = \dfrac{x^5 - x}{1 + x^2}$

Homework:

27, 31, 35, 59 (b-d-f) pages (25- 26)

*)graphs of fuctions :

$f(x) + c \longrightarrow$ Shifting up by (c)

$f(x) - c \longrightarrow$ Shifting down by (c)

$f(x + c) \longrightarrow$ Shifting left by (c)

$f(x - c) \longrightarrow$ Shifting right by (c)

$-f(x) \longrightarrow$ Reflection about x-axis

$f(-x) \longrightarrow$ Reflection about y-axis

$af(x) \longrightarrow$ Vertical stretching

$\frac{1}{a} f(x) \longrightarrow$ Vertical compressing

$f(ax) \longrightarrow$ Horizontal compressing

$f(\frac{1}{a} x) \longrightarrow$ Horizontal streaching