IQ ACADEMY

Exercise set (4.3):

1(b)-2(b)-3(a)-4(a)-7(a)-15-20-22-28-33

1–12 Evaluate the integrals using the indicated substitutions.

1. (a) $\int 2x(x^2+1)^{23}\,dx;\ u = x^2+1$

(b) $\int \cos^3 x \sin x\,dx;\ u = \cos x$

b) $u = \cos x$

$du = -\sin x \longrightarrow -du = \sin x$

$\int (u^3) \cdot -du = -\int u^3\,du$

$= -\dfrac{u^{3+1}}{3+1} + C$

$= -\dfrac{u^4}{4} + C$

$= -\dfrac{\cos^4 x}{4} + C$

2. (b) $\int \frac{3x\,dx}{\sqrt{4x^2+5}}$; $u = 4x^2 + 5$

$u = 4x^2 + 5$

$du = 8x\,dx \longrightarrow \frac{du}{8} = x\,dx$

$= 3\int \frac{du}{8\sqrt{u}} = \frac{3}{8}\int \frac{du}{\sqrt{u}}$

$= \frac{3}{8} \cdot 2\sqrt{u} + C$

$= \frac{6}{8}\sqrt{4x^2+5} + C$

$= \frac{3}{4}\sqrt{4x^2+5} + C$

3. (a) $\displaystyle\int \sec^2(4x+1)\,dx;\ u = 4x+1$

$du = 4\,dx \longrightarrow \dfrac{du}{4} = dx$

$= \displaystyle\int \sec^2(u)\,\frac{du}{4} = \frac{1}{4}\int \sec^2(u)\,du$

$= \dfrac{1}{4}\tan u + C$

$= \dfrac{1}{4}\tan(4x+1) + C$

4. (a) $\int \sqrt{\sin \pi\theta} \cos \pi\theta \, d\theta; \; u = \sin \pi\theta$

$u = \sin \pi \theta$

$du = \pi \cos(\pi\theta)\, d\theta \rightarrow \frac{du}{\pi} = \cos(\pi\theta)\, d\theta$

$\int \sqrt{u} \, \frac{du}{\pi} = \frac{1}{\pi} \int u^{1/2} \, du$

$= \frac{1}{\pi} \cdot \frac{u^{1/2+1}}{1/2+1} + C$

$= \frac{1}{\pi} \cdot \frac{u^{3/2}}{3/2} + C$

$= \frac{1}{\pi} \cdot \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3\pi} u^{3/2} + C$

$= \frac{2(\sin \pi\theta)^{3/2}}{3\pi}$

7. (a) $\int x^2\sqrt{1+x}\,dx;\ \ u = 1 + x$

(b) $\int [\csc(\sin x)]^2 \cos x\,dx;\ \ u = \sin x$

a) $u = 1 + X$

$du = dX$

$u = 1 + X \longrightarrow u - 1 = X \longrightarrow (u-1)^2 = X^2$

$\int (u-1)^2 \sqrt{u}\, du$

$= \int (u^2 - 2u + 1)(u^{1/2})\, du = \int (u^{2+1/2} - 2u^{1+1/2} + u^{1/2})\, du$

$= \int (u^{5/2} - 2u^{3/2} + u^{1/2})\, du$

$= \dfrac{u^{5/2+1}}{5/2+1} - 2\dfrac{u^{3/2+1}}{3/2+1} + \dfrac{u^{1/2+1}}{1/2+1} + C$

$= \dfrac{u^{7/2}}{7/2} - 2\dfrac{u^{5/2}}{5/2} + \dfrac{u^{3/2}}{3/2} + C$

$= \dfrac{2}{7}u^{7/2} - 2\left(\dfrac{2}{5}\right)u^{5/2} + \dfrac{2}{3}u^{3/2} + C$

$= \dfrac{2}{7}u^{7/2} - \dfrac{4}{5}u^{5/2} + \dfrac{2}{3}u^{3/2} + C$

$= \dfrac{2}{7}(1+X)^{7/2} - \dfrac{4}{5}(1+X)^{5/2} + \dfrac{2}{3}(1+X)^{3/2} + C$

11–36 Evaluate the integrals using appropriate substitutions.

15. $\int \sec 4x \tan 4x \, dx$

$u = 4X$

$du = 4dX \longrightarrow \frac{du}{4} = dX$

$\int \sec(u)\tan(u)\frac{du}{4}$

$= \frac{1}{4}\int \sec(u)\tan(u)\,du$

$= \frac{1}{4}\sec(u) + C$

$= \frac{1}{4}\sec(4X) + C$

20. $\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$

$u = X^3 + 3X$

$du = 3X^2 + 3\,dX \longrightarrow du = 3(X^2+1)dX$

$\frac{du}{3} = (X^2+1)dX$

$\frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int \frac{du}{u^{1/2}}$

$= \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}\cdot\frac{u^{-1/2+1}}{-1/2+1} + C$

$= \frac{1}{3}\cdot\frac{u^{1/2}}{\frac{1}{2}} + C$

$= \frac{1}{3}\cdot\frac{2}{1}\,u^{1/2} + C$

$= \frac{2}{3}\sqrt{u} + C$

$= \frac{2}{3}\sqrt{X^3+3X} + C$

22. $\int \frac{\sin(1/x)}{3x^2}\,dx$

$u = 1/X$

$du = -\frac{1}{X^2}\,dX \longrightarrow -du = \frac{1}{X^2}\,dX$

$-\frac{1}{3}\int \sin(u)\,du$

$= -\frac{1}{3}(-\cos u) + C$

$= \frac{1}{3}\cos(1/X) + C$

28. $\displaystyle\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4}\,d\theta$

$u = 1 + 2\sin 4\theta$

$du = 8\cos 4\theta \, d\theta \rightarrow \frac{du}{8} = \cos 4\theta \, d\theta$

$\frac{1}{8}\int \frac{du}{u^4}$

$= \frac{1}{8}\int u^{-4}\,du$

$= \frac{1}{8}\cdot\frac{u^{-3}}{-3} + C$

$= -\frac{1}{24u^3} + C$

$= -\frac{1}{24(1+2\sin 4\theta)^3} + C$

33. $\displaystyle\int \frac{y}{\sqrt{2y+1}}\,dy$

$u = 2Y + 1$

$du = 2\,dY \longrightarrow \dfrac{du}{2} = dY$

$u = 2Y + 1 \longrightarrow u - 1 = 2Y$

$Y = \dfrac{u-1}{2}$

$\displaystyle\int \frac{u-1}{2\sqrt{u}} \cdot \frac{du}{2}$

$\displaystyle = \frac{1}{4}\int \frac{u-1}{u^{1/2}}\,du$

$\displaystyle = \frac{1}{4}\int \frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\,du$

$\displaystyle = \frac{1}{4}\int u^{1/2} - u^{-1/2}\,du$

$\displaystyle = \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right) + C$

$\displaystyle = \frac{1}{4}\cdot\frac{2}{3}\,u^{3/2} - \frac{1}{4}\cdot 2\,u^{1/2} + C$

$\displaystyle = \frac{2}{12}\,u^{3/2} - \frac{2}{4}\,u^{1/2} + C$

$\displaystyle = \frac{1}{6}\,u^{3/2} - \frac{1}{2}\,u^{1/2} + C$

$$= \frac{1}{6}(2Y+1)^{3/2} - \frac{1}{2}(2Y+1)^{1/2} + C$$