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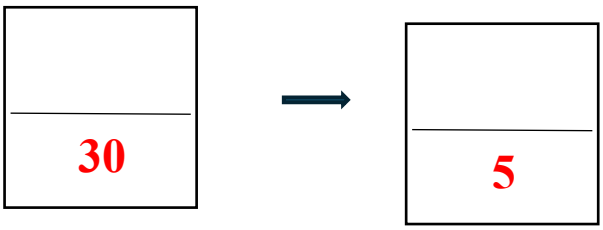
Worksheet 1 - Chapter 0- Math101



Group: ……

Name of Members	Questions Solved

Question 1 : (12 marks)

A- Find the domain following functions

1) $f(x) = \sqrt{3x+6} + \frac{1}{\sqrt{x-1}}$

2) $f(x) = \frac{x^2-9}{x+3} - \sqrt{x-2}$

3) $f(x) = cos\,(3x-5)$

B- Find the range following functions

1) $f(x) = 4sin\,x + 7$

2) $f\,(x) = \frac{7}{2}\sqrt{x^2-3}$

3) $f(x) = \frac{x+2}{3x-2}$

A- Find the domain following functions

1) $f(x) = \sqrt{3x+6} + \frac{1}{\sqrt{x-1}}$

$3X+6 \geq 0 \rightarrow 3X \geq -6$

$\rightarrow X \geq -6/3 \rightarrow X \geq -2 = [-2, +\infty)$

$X-1 > 0 \rightarrow X > 1 = (1, +\infty)$

$D_f = [-2, +\infty) \cap (1, +\infty)$

$D_f = (1, +\infty)$

2) $f(x) = \frac{x^2-9}{x+3} - \sqrt{x-2}$

$X+3 \neq 0 \rightarrow X \neq -3 \rightarrow (-\infty, -3) \cup (-3, +\infty)$

$X-2 \geq 0 \rightarrow X \geq 2 = [2, +\infty)$

$D_f = ((-\infty, -3) \cup (-3, +\infty)) \cap [2, +\infty)$

$D_f = [2, +\infty)$

3) $f(x) = cos\,(3x-5)$

domain of Cos is $(-\infty, +\infty)$

$(3X-5)$ is Polynomial so the domain is $(-\infty, +\infty)$

$D_f = (-\infty, +\infty)$

B- Find the range following functions

1) $f(x) = 4\sin x + 7$

range of $\sin x \rightarrow -1 \leq \sin x \leq 1$

$-1 \leq \sin x \leq 1$

$-4 \leq 4\sin x \leq 4$

$-4+7 \leq 4\sin x + 7 \leq 4+7$

$3 \leq 4\sin x + 7 \leq 11$

$R_f = [3, 11]$

2) $f(x) = \frac{7}{2}\sqrt{x^2 - 3}$

$x^2 - 3 \geq 0 \rightarrow x^2 \geq 3 \rightarrow \sqrt{x^2} \geq \sqrt{3} \rightarrow |x| \geq \sqrt{3}$

$x \geq \sqrt{3}$ or $x \leq -\sqrt{3}$

$D = (-\infty, -\sqrt{3}\,] \cup [\sqrt{3}, +\infty)$

Range:

$\sqrt{x^2 - 3} \geq 0$

$\frac{7}{2}\sqrt{x^2 - 3} \geq 0 \cdot \frac{7}{2}$

$\frac{7}{2}\sqrt{x^2 - 3} \geq 0$

$R_f = [0, +\infty)$

3) $f(x) = \frac{x+2}{3x-2}$

$Y = \frac{x+2}{3x-2}$

$Y(3x-2) = x+2$

$3xY - 2Y = x+2$

$3xY - 2Y - 2 = x \rightarrow -2Y - 2 = x - 3xY$

$-2Y - 2 = x(1-3Y) \rightarrow x = \frac{-2Y-2}{1-3Y}$

$x = -\frac{2Y+2}{1-3Y} \rightarrow$

$f^{-1}(x) = -\frac{2x+2}{1-3x}$

$D_{f^{-1}} \rightarrow 1-3x \neq 0$

$1 \neq 3x \rightarrow x \neq 1/3 \rightarrow (-\infty, 1/3) \cup (1/3, +\infty)$

$R_f = (-\infty, 1/3) \cup (1/3, +\infty)$

Question 2 : (4marks)

Let $f(x) = \frac{3}{x^2-1}$, $g(x) = \frac{x^2-1}{x}$

Find the functions of fg, f/g and their domains

Question 3: (4 marks)

Determine which of the following functions are odd, even, or neither

A-$f(x) = \frac{|x-5|}{x}$

B-$f(x) = \frac{2x^4+3x^2}{x^3+3x}$

Question 2 : (4marks)

Let $f(x) = \frac{3}{x^2-1}$, $g(x) = \frac{x^2-1}{x}$

Find the functions of fg, f/g and their domains

$$fg = \frac{3}{X^2-1} \cdot \frac{X^2-1}{X} = \frac{3}{X}$$

$$f/g = \frac{3}{X^2-1} \Big/ \frac{X^2-1}{X} = \frac{3}{X^2-1} \cdot \frac{X}{X^2-1} = \frac{3X}{(X^2-1)^2}$$

$D_f = (X^2-1) = (X-1)(X+1)$

$X-1 \neq 0 \rightarrow X \neq 1$

$X+1 \neq 0 \rightarrow X \neq -1$

$= (-\infty,-1) \cup (-1,1) \cup (1,+\infty)$

$D_g = X \neq 0 \rightarrow (-\infty,0) \cup (0,+\infty)$

$D_{f/g} = D_f \cap D_g = ((-\infty,-1) \cup (-1,1) \cup (1,+\infty)) \cap ((-\infty,0) \cup (0,+\infty))$

$= (-\infty,-1) \cup (-1,0) \cup (0,1) \cup (1,+\infty)$

$D_{fg} = D_f \cap D_g = (-\infty,-1) \cup (-1,0) \cup (0,1) \cup (1,+\infty)$

Question 3: (4 marks)

Determine which of the following functions are odd, even, or neither

A- $f(x) = \frac{|x-5|}{x}$

B- $f(x) = \frac{2x^4+3x^2}{x^3+3x}$

a) $f(-x) = \frac{|(-x)-5|}{-x} = \frac{|-(x+5)|}{-x} = \frac{|x+5|}{-x} \neq f(x)$ (not even)

$f(-x) = \frac{|x+5|}{-x} \neq -f(x)$ (not odd)

So its neither

b) $f(-x) = \frac{2(-x)^4+3(-x)^2}{(-x)^3+3(-x)} = \frac{2x^4+3x^2}{-x^3-3x} \neq f(x)$ (not even)

$f(-x) = \frac{2x^4+3x^2}{-x^3-3x} = -\frac{2x^4+3x^2}{x^3+3x} = -f(x)$ → odd function

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} = -\left(\frac{a}{b}\right)$$

Question 4: (6 marks)

For the function $f(x) = \sqrt{3x-1}$ +5

A-Find the formula of $f^{-1}(x)$

B- State the range of $f^{-1}(x)$

C-Evaluate $f\left(\frac{5}{3}\right)$ *and* $f^{-1}(5)$

Question 4: (6 marks)

For the function $f(x) = \sqrt{3x-1} + 5$

A-Find the formula of $f^{-1}(x)$

B- State the range of $f^{-1}(x)$

C-Evaluate $f\left(\frac{5}{3}\right)$ and $f^{-1}(5)$

a) $Y = \sqrt{3X-1} + 5$

$(Y-5)^2 = (\sqrt{3X-1})^2$

$(Y-5)^2 = 3X-1$

$(Y-5)^2 + 1 = 3X$

$X = \frac{(Y-5)^2 + 1}{3}$

$f^{-1}(X) = \frac{(X-5)^2 + 1}{3}$ '

b) $D_f = R_{f^{-1}}$

$D_f = 3X-1 \geq 0 \rightarrow 3X \geq 1$

$X \geq 1/3 = [1/3, +\infty)$

c) $f(x) = \sqrt{3x - 1} + 5$

$$f\left(\frac{5}{3}\right) = \sqrt{3\left(\frac{5}{3}\right) - 1} + 5$$

$$= \sqrt{5 - 1} + 5 = 2 + 5 = 7$$

$$f^{-1}(x) = \frac{(x - 5)^2 + 1}{3}$$

$$f^{-1}(5) = \frac{(5 - 5)^2 + 1}{3} = \frac{1}{3}$$

Question 5: (4 marks)

Let $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = 5x - 1$

Find the functions of $f o g$ and state the domain of the composition

Question 5: (4 marks)

Let $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = 5x - 1$

Find the functions of $f o g$ and state the domain of the composition

$$(fog)(x) = f(g(x)) = f(5x-1)$$

$$= \frac{1}{\sqrt{5x-1}}$$

$$D_g = (-\infty, +\infty)$$

$$D_{\sqrt{5x-1}} = 5x - 1 > 0 \rightarrow 5x > 1$$

$$x > 1/5 = (1/5, +\infty)$$

$$D_{fog} = D_g \cap D_{\sqrt{5x-1}} = (-\infty, +\infty) \cap (1/5, +\infty)$$

$$= (1/5, +\infty)$$