0.4 INVERSE FUNCTIONS

INVERSE FUNCTIONS

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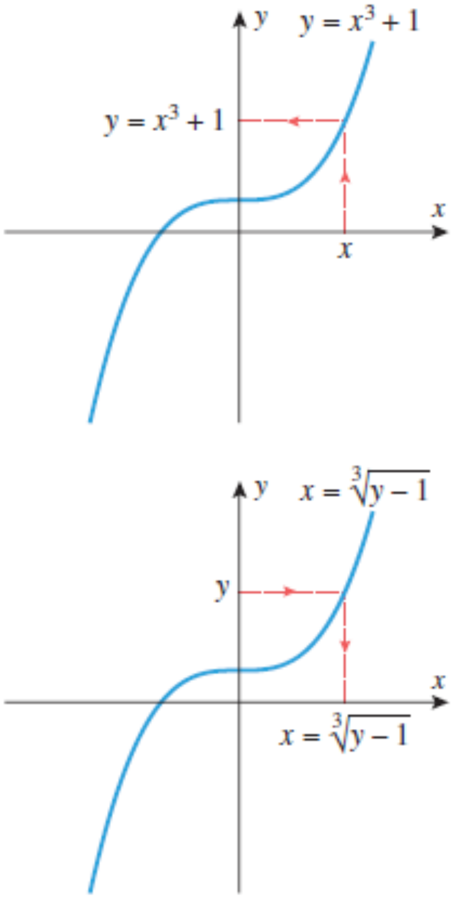

▲ Figure 0.4.1

الشروط تحقق

0.4.1 DEFINITION If the functions f and g satisfy the two conditions

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$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f$$
$$f(g(y)) = y \text{ for every } y \text{ in the domain of } g$$

then we say that ***f* is an inverse of *g*** and ***g* is an inverse of *f*** or that ***f* and *g* are inverse functions**.

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▶ **Example 1** The computations in (1) show that $g(y) = \sqrt[3]{y-1}$ is the inverse of $f(x) = x^3 + 1$. Thus, we can express g in inverse notation as

$$f^{-1}(y) = \sqrt[3]{y-1}$$

and we can express the equations in Definition 0.4.1 as

$$\begin{aligned} f^{-1}(f(x)) &= x \quad \text{for every } x \text{ in the domain of } f \\ f(f^{-1}(y)) &= y \quad \text{for every } y \text{ in the domain of } f^{-1} \end{aligned} \tag{2}$$

We will call these the ***cancellation equations*** for f and f^{-1}. ◀

WARNING

If f is a function, then the -1 in the symbol f^{-1} always denotes an inverse and *never* an exponent. That is,

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$$f^{-1}(x) \quad \textit{never means} \quad \frac{1}{f(x)}$$

المتغير المستقل

CHANGING THE INDEPENDENT VARIABLE

$$\begin{aligned} f^{-1}(f(x)) &= x \quad \text{for every } x \text{ in the domain of } f \\ f(f^{-1}(x)) &= x \quad \text{for every } x \text{ in the domain of } f^{-1} \end{aligned} \tag{3}$$

▸ **Example 2** تأكد Confirm each of the following.

(a) The inverse of $f(x) = 2x$ is $f^{-1}(x) = \frac{1}{2}x$.

(b) The inverse of $f(x) = x^3$ is $f^{-1}(x) = x^{1/3}$.

Solution (a)

Solution (b)

DOMAIN AND RANGE OF INVERSE FUNCTIONS

$$\begin{aligned}\text{domain of } f^{-1} &= \text{range of } f \\ \text{range of } f^{-1} &= \text{domain of } f\end{aligned} \qquad (4)$$

A METHOD FOR FINDING INVERSE FUNCTIONS

0.4.2 THEOREM *If an equation $y = f(x)$ can be solved for x as a function of y, say $x = g(y)$, then f has an inverse and that inverse is $g(y) = f^{-1}(y)$.*

Theorem 0.4.2 provides us with the following procedure for finding the inverse of a function.

A Procedure for Finding the Inverse of a Function f

Step 1. Write down the equation $y = f(x)$.

Step 2. If possible, solve this equation for x as a function of y.

Step 3. The resulting equation will be $x = f^{-1}(y)$, which provides a formula for f^{-1} with y as the independent variable.

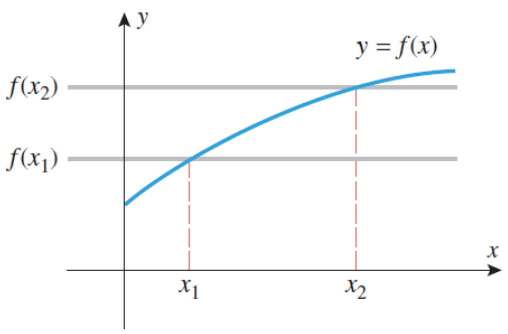
Step 4. If y is acceptable as the independent variable for the inverse function, then you are done, but if you want to have x as the independent variable, then you need to interchange x and y in the equation $x = f^{-1}(y)$ to obtain $y = f^{-1}(x)$.

▶ **Example 4** Find a formula for the inverse of $f(x) = \sqrt{3x - 2}$ with x as the independent variable, and state the domain of f^{-1}.

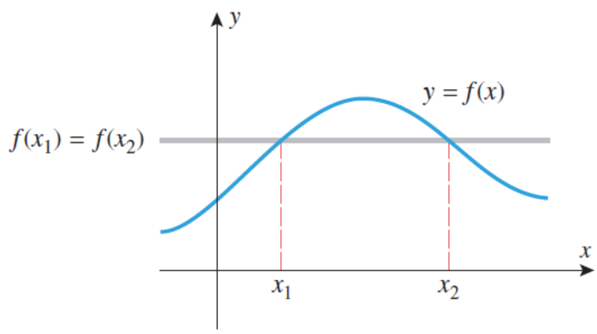
Solution.

EXISTENCE OF INVERSE FUNCTIONS

0.4.3 THEOREM *A function has an inverse if and only if it is one-to-one.*



One-to-one, since $f(x_1) \neq f(x_2)$ if $x_1 \neq x_2$



Not one-to-one, since $f(x_1) = f(x_2)$ and $x_1 \neq x_2$

اختبار الافقي

> **0.4.4 THEOREM** (***The Horizontal Line Test***) *A function has an inverse function if and only if its graph is cut at most once by any horizontal line.*

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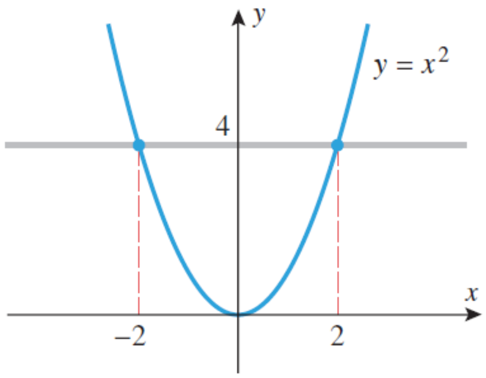
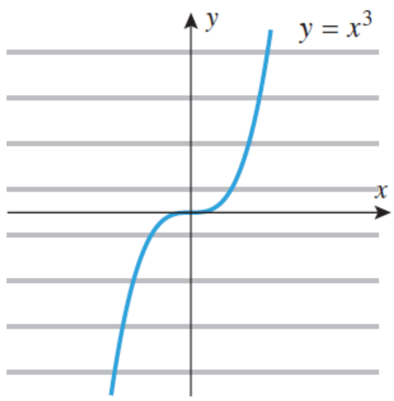
▶ **Example 5** Use the horizontal line test to show that $f(x) = x^2$ has no inverse but that $f(x) = x^3$ does.



▲ **Figure 0.4.4**



▲ **Figure 0.4.5**

Homework:

1 (a-b), 3 (a-c),10, 11, 17, 18 pages (44- 45)