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Revision

Math 101T

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Sets and Elements

- Set in mathematics, is an organized collection of objects.
- Usually, sets are represented in curly braces $\{\}$, for example, $A = \{1, 2, 3, 4\}$.
- The notation $a \in A$ means that a is an element of the set A, and $a \notin A$ means that a is NOT an element of the set A

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Set Operations

Union of Sets

For two given sets A and B, $A \cup B$ (read as A union B) is the set of all elements that are in either set.

Intersection of Sets

For two given sets A and B, $A \cap B$ (read as A intersection B) is the set of common elements that belong to set A and B.

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Set of Numbers

Common Sets

- The set of **Natural** numbers is denoted by $\mathbb{N}$,

  $\mathbb{N} = \{1, 2, 3, \dots\}$

- The set of **Whole** numbers is denoted by $\mathbb{W}$,

  $\mathbb{W} = \{0, 1, 2, 3, \dots\}$

- The set of **Integer** numbers is denoted by $\mathbb{Z}$,

  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

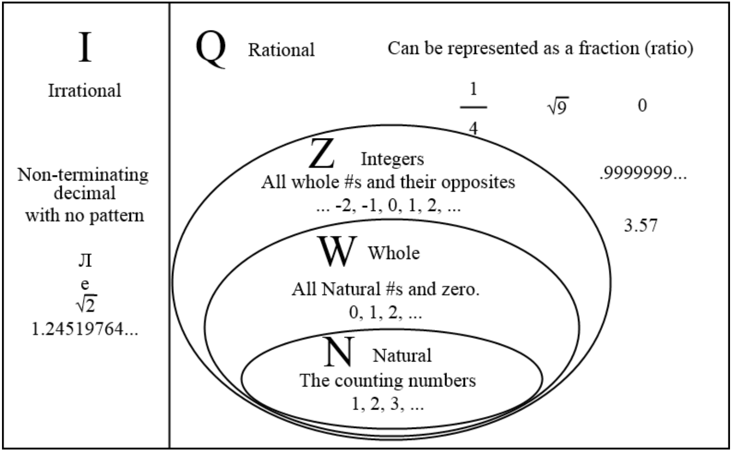
Set of Numbers

Common Sets

- The set of **Rational** numbers is denoted by $\mathbb{Q}$,
  $\mathbb{Q} = \left\{ \frac{p}{q},\ p, q \in \mathbb{Z},\ q \neq 0 \right\}$
- The numbers that cannot be expressed as the ratio of two integers is called irrational numbers. For example the numbers $\sqrt{2},\ \pi$ are NOT rational numbers. The set of **Irrational** numbers is denoted by $\mathbb{I}$
- The set of **Real** numbers is denoted by $\mathbb{R}$,
  $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$

Set of Numbers

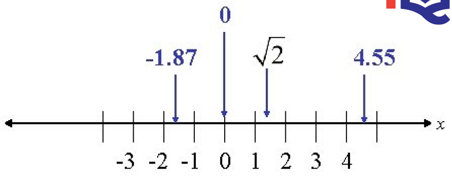
Real Numbers



Introduction to Inequalities and Interval Notation

We Study how to solve inequalities which is the process of finding the set of numbers that makes the inequality a true statement.

The real numbers can be illustrated geometrically with a diagram called a **number line**. Each real number corresponds to exactly one point on the line and vice versa.



Introduction to Inequalities and Interval Notation

The solution of an inequality can be represented in interval notation

Intervals

An interval is a set of real numbers that contains all real numbers lying between two given numbers. If $a, b \in \mathbb{R}$ and $a < b$, then we have the following different type of intervals.

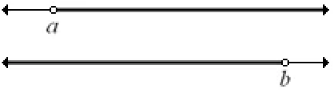
- The **open interval** $(a, b) = \{x;\ a < x < b\}$

- The **closed interval** $[a, b] = \{x;\ a \leq x \leq b\}$

Intervals

- The **half-open interval** $(a, b] = \{x;\ a < x \leq b\}$ and the **half-closed interval** $[a, b) = \{x;\ a \leq x < b\}$

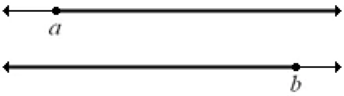
- The **infinite-open interval** $(a, \infty) = \{x;\ x > a\}$ or $(-\infty, b) = \{x;\ x < b\}$



Introduction to Inequalities and Interval Notation

Intervals

- The **infinite-closed interval** $[a, \infty) = \{x;\ x \geq a\}$ or $(-\infty, b] = \{x;\ x \leq b\}$



- The set of all **Real numbers** can be expressed in interval notation as $\mathbb{R} = (-\infty, \infty)$

Properties of inequalities *If* $a, b, c \in \mathbb{R}$, *then*

1. If $a < b$ then $a + c < b + c$ and $a - c < b - c$
2. If $a < b$ and $b < c$ then $a < c$
3. If $a < b$ and $c > 0$ then $a\,c < b\,c$ and $\dfrac{a}{c} < \dfrac{b}{c}$
4. If $a < b$ and $c < 0$ then $a\,c > b\,c$ and $\dfrac{a}{c} > \dfrac{b}{c}$
5. If $0 < a < b$ then $\dfrac{1}{a} > \dfrac{1}{b}$
6. If $a < b < 0$ then $\dfrac{1}{a} > \dfrac{1}{b}$

Linear inequalities

A Linear inequality is an inequality which involves a linear. Suppose $a \neq 0$, then a Linear inequality can be written $ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$ or $ax + b \leq 0$.

Introduction to Inequalities and Interval Notation

Example (1)

Solve the following inequalities

a. $x + 5 < 3$ → $X+5-5 < 3-5$ → $X < -2 = (-\infty, -2)$

b. $2 - x < 7$ → $-2+X > -7$ → $-2+X+2 > -7+2$ → $X > -5 = (-5, +\infty)$

c. $x + 5 < 3$ and $2 - x < 7$ → $(-\infty, -2) \cap (-5, +\infty)$ → $(-5, -2)$

d. $x + 5 < 3$ or $2 - x < 7$ → $(-\infty, -2) \cup (-5, +\infty)$ → $(-\infty, +\infty)$

e. $-2x - 1 < 5$ → $-2X-1+1 < 5+1$ → $-2X < 6$ → $\frac{-2X}{-2} < \frac{6}{-2}$ → $X > -3$ $= (-3, +\infty)$

f. $3x - 15 \geq 2x + 11$ → $3X-15-2X \geqslant 2X+11-2X$ → $X-15 \geqslant 11$ → $X \geqslant 26$ $= [26, +\infty)$

g. $-5 \leq 3 - 4x < 15$ → $-8 \leqslant -4X < 12$ → $2 \geqslant X > -3$ → $-3 < X \leqslant 2 = (-3, 2]$

h. $x \neq \sqrt{2}$ $= (-\infty, \sqrt{2}) \cup (\sqrt{2}, +\infty)$

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inequalities with absolute value

The absolute value (or modulus) of a real number x is the non-negative value of x without regard to its sign. For example, the absolute value of 2 is 2, and the absolute value of -2 is also 2.

The absolute value of a number describes the distance between a number and zero on a number line

Definition

The absolute value of a real number $|x|$ is defined by

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example (2)

Rewrite each expression without absolute value

$|\sqrt{2}-1|$, $|3-\pi|$, $|3x-1|$

Properties of absolute value

If a and b are real numbers, then

1. $|a| \geq 0$
2. $|ab| = |a|\,|b|$
3. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \qquad b \neq 0$
4. $|a+b| \leq |a| + |b|$

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Example (2)

Rewrite each expression without absolute value

$|\sqrt{2}-1|$, $|3-\pi|$, $|3x-1|$

① $|\sqrt{2}-1|$

$\sqrt{2}=1.414$

$\sqrt{2}-1=0.414>0$

$|\sqrt{2}-1|=\sqrt{2}-1$

② $|3-\pi|$

$\pi=3.141$

$3-\pi=-0.141$

$|3-\pi|=\pi-3$

③ $|3x-1|$

$3x-1\geqslant 0$

$3x\geqslant 1 \rightarrow x\geqslant \frac{1}{3}$

$|3x-1|=3x-1$

$3x-1<0$

$3x<1 \rightarrow x<\frac{1}{3}$

$|3x-1|=1-3x$

Properties of absolute value

If a and b are real numbers, then

5. $\sqrt{a^2} = |a|$
6. If $a > 0$, then $|x| < a$ ($|x| \leq a$) if and only if $-a < x < a$ ($-a \leq x \leq a$)
7. If $a > 0$, then $|x| > a$ ($|x| \geq a$) if and only if $x > a$ or $x < -a$ ($x \geq a$ or $x \leq -a$)
8. $|a| \leq |b|$ if and only if $a^2 \leq b^2$

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Example (3)

Solve the following inequalities

a. $|x-5|<3$ → $-3<x-5<3$ → $2<x<8 = (2,8)$

b. $|3x+2|<6$ → $-6<3x+2<6$ → $-8<3x<4$ → $-\frac{8}{3}<x<\frac{4}{3} = (-\frac{8}{3},\frac{4}{3})$

c. $|3-2x|\geq 7$ → $3-2x\geq 7$ → $-2x\geq 4$ → $x\leq -2$ → $(-\infty,-2]$
→ $3-2x\leq -7$ → $-2x\leq -10$ → $x\geq 5$ → $[5,+\infty)$
$(-\infty,-2]\cup[5,+\infty)$

d. $|x+4|<-1$

No Solution

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Introduction to Inequalities and Interval Notation

Quadratic and rational inequalities

Example (4)

Solve the following inequalities

a. $x^2 \leq 9 \rightarrow \sqrt{x^2} \leqslant \sqrt{9} \rightarrow |x| \leqslant 3 \rightarrow -3 \leqslant x \leqslant 3 \rightarrow [-3,3]$

b. $x^2 - 3x < -2 \rightarrow x^2 - 3x + 2 < 0 \rightarrow (x-1)(x-2) < 0 \rightarrow (1,2)$

c. $\dfrac{2x-3}{x+1} > 1$

d. $\dfrac{4}{x+5} < \dfrac{3}{x-2}$

e. $\dfrac{1}{x^2+4} \geq 0$

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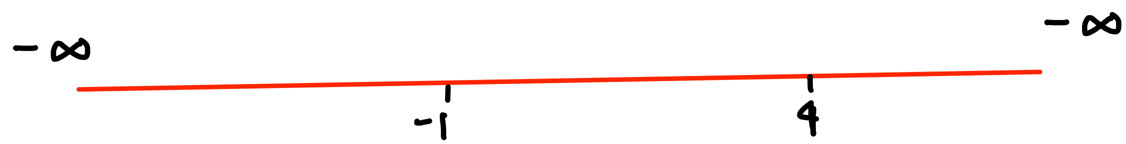
c. $\dfrac{2x-3}{x+1} > 1 \rightarrow \dfrac{2x-3}{x+1} - 1 > 0 \rightarrow \dfrac{2x-3-(x+1)}{x+1} > 0$

$$\frac{x-4}{x+1} > 0$$

$x - 4 > 0 \rightarrow x > 4$

$x + 1 > 0 \rightarrow x > -1$



the interval $= (-\infty, -1) \cup (4, +\infty)$

	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
$x-4$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
$\dfrac{x-4}{x+1}$	$+$	$-$	$+$

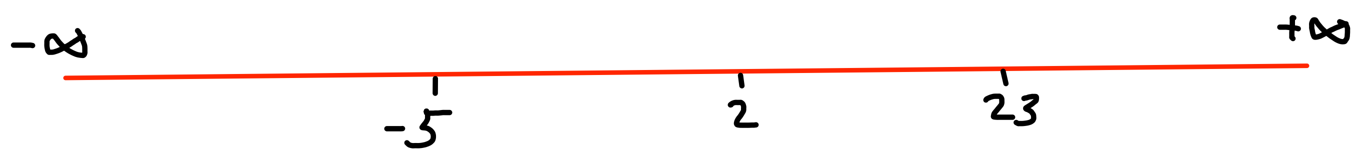
d. $\frac{4}{x+5} < \frac{3}{x-2} \rightarrow \frac{4}{X+5} - \frac{3}{X-2} < 0$

$$\frac{4(X-2)-3(X+5)}{(X+5)(X-2)} < 0 \rightarrow \frac{X-23}{(X+5)(X-2)} < 0$$

$X-23<0 \rightarrow X<23$

$X+5<0 \rightarrow X<-5$

$X-2<0 \rightarrow X<2$



the interval $= (-\infty,-5) \cup (2,23)$

	$(-\infty,-5)$	$(-5,2)$	$(2,23)$	$(23,+\infty)$
$X-23$	−	−	−	+
$X+5$	−	+	+	+
$X-2$	−	−	+	+
$\frac{X-23}{(X+5)(X-2)}$	−	+	−	+

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e. $\frac{1}{x^2+4} \geq 0$

$x^2+4>0 \rightarrow x^2>-4 \rightarrow$ the inteval $=(-\infty, +\infty) = R$

Trigonometric Properties

Trigonometric Table

Angle (Radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Remark

Try to remember the definition of the following

$\tan x = \frac{\sin(x)}{\cos(x)}$ $\quad \cot x = \frac{\cos x}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \csc x = \frac{1}{\sin x}$

Trigonometric Properties

Trigonometric Identities

- $\cos^2\theta + \sin^2\theta = 1$
- $\cos(-\theta) = \cos\theta$ لان Cos دالة زوجية
- $\sin(-\theta) = -\sin\theta$ لان Sin دالة فردية

Trigonometric Zeros

- $\sin\theta = 0$ if and only if $\theta = n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$
- $\cos\theta = 0$ if and only if $\theta = \dfrac{\pi}{2} + n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Important Formulas

COMMON FACTORING EXAMPLES

- $a^2 - b^2 = (a-b)(a+b)$
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$

The Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Important Formulas

ARITHMETIC OPERATIONS EXAMPLES

- $ab + ac = a(b + c)$
- $\dfrac{a}{b} \pm \dfrac{c}{d} = \dfrac{ad \pm bc}{bd}, \quad b, d \neq 0$
- $\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{ac}{bd}, \quad b, d \neq 0$
- $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \times \dfrac{d}{c} = \dfrac{ad}{bc}, \quad b, c, d \neq 0$