حساب التفاضل والتكامل 1- ريض 101ت

Calculus -Masc 101T

Text Book:

H. Anton, I. Bivens, and S. Davis,

Calculus: Late Transcendental Single and multivariable, 9th Edition

Chapters

0,1,2,3,4,6

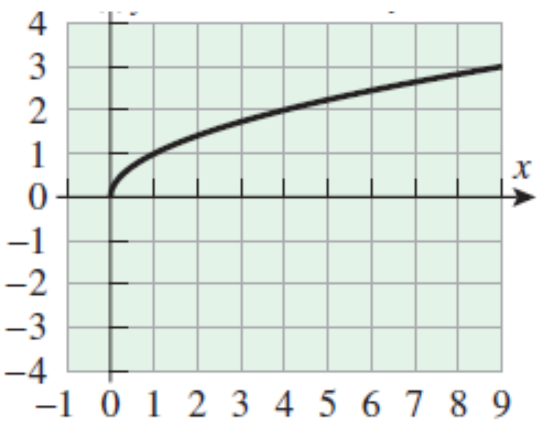
0.1 Functions

> **0.1.1 DEFINITION** If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that ***y is a function of x***.

Four common methods for representing functions are:

- **Numerically by tables**

- **Geometrically by graphs**



- **Algebraically by formulas**

- **Verbally**

0.1.2 DEFINITION A ***function*** f is a rule that associates a unique output with each input. If the input is denoted by x, then the output is denoted by $f(x)$ (read "f of x").

INDEPENDENT AND DEPENDENT VARIABLES

For a given input x, the output of a function f is called the ***value*** of f at x or the ***image*** of x under f.

Sometimes we will want to denote the output by a single letter, say y, and write

$$y = f(x)$$

- This equation expresses y as a function of x;
- The variable x is called the ***independent variable*** (or ***argument***) of f
- The variable y is called the ***dependent variable*** of f.
- If the independent and dependent variables are real numbers, in which case we say that f is a ***real-valued function of a real variable***.

Table 0.1.2

x	0	1	2	3
y	3	4	-1	6

Example 1 Table 0.1.2 describes a functional relationship $y = f(x)$ for which

$f(0) = 3$ — f associates $y = 3$ with $x = 0$.

$f(1) = 4$ — f associates $y = 4$ with $x = 1$.

$f(2) = -1$ — f associates $y = -1$ with $x = 2$.

$f(3) = 6$ — f associates $y = 6$ with $x = 3$.

Example 2 The equation

$$y = 3x^2 - 4x + 2$$

has the form $y = f(x)$ in which the function f is given by the formula

$$f(x) = 3x^2 - 4x + 2$$

0.1.3 THE VERTICAL LINE TEST *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*

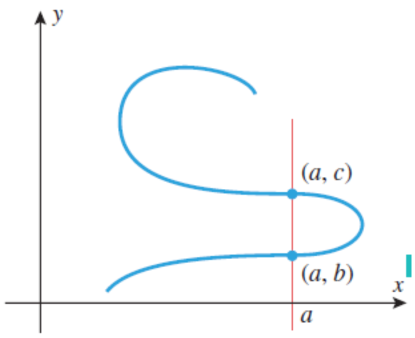
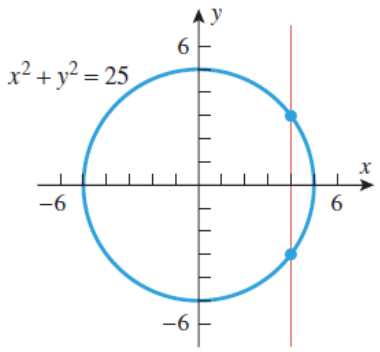


Figure 0.1.7 This curve cannot be the graph of a function.

▶ **Example 3** The graph of the equation

$$x^2 + y^2 = 25$$



▲ **Figure 0.1.8**

THE ABSOLUTE VALUE FUNCTION

Recall that the ***absolute value*** or ***magnitude*** of a real number x is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative. Thus,

$$|5| = 5, \quad \left|-\tfrac{4}{7}\right| = \tfrac{4}{7}, \quad |0| = 0$$

0.1.4 PROPERTIES OF ABSOLUTE VALUE *If a and b are real numbers, then*

(a) $|-a| = |a|$ — A number and its negative have the same absolute value.

(b) $|ab| = |a|\,|b|$ — The absolute value of a product is the product of the absolute values.

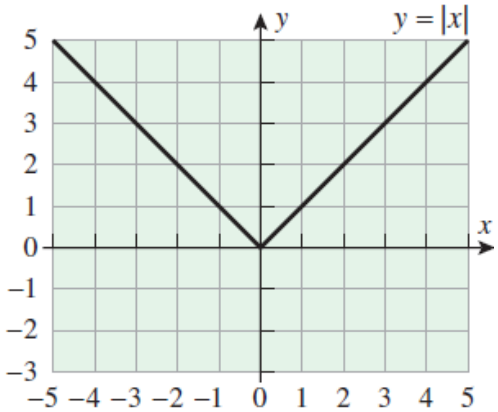
(c) $|a/b| = |a|/|b|, b \neq 0$ — The absolute value of a ratio is the ratio of the absolute values.

(d) $|a + b| \leq |a| + |b|$ — The ***triangle inequality***

The graph of the function $f(x) = |x|$ can be obtained by graphing the two parts of the equation

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

separately. Combining the two parts produces the V-shaped graph in Figure 0.1.9.



▲ Figure 0.1.9

Remark:

PIECEWISE-DEFINED FUNCTIONS

The absolute value function $f(x) = |x|$ is an example of a function that is defined ***piecewise*** in the sense that the formula for f changes, depending on the value of x.

▶ Example

8. Find $g(3)$, $g(-1)$, $g(\pi)$, $g(-1.1)$, and $g(t^2 - 1)$.

$$\text{(b)}\quad g(x) = \begin{cases} \sqrt{x+1}, & x \geq 1 \\ 3, & x < 1 \end{cases}$$

Solution.

DOMAIN AND RANGE

If x and y are related by the equation $y = f(x)$, then the set of all allowable inputs (x-values) is called the ***domain*** of f, and the set of outputs (y-values) that result when x varies over the domain is called the ***range*** of f.

0.1.5 DEFINITION If a real-valued function of a real variable is defined by a formula, and if no domain is stated explicitly, then it is to be understood that the domain consists of all real numbers for which the formula yields a real value. This is called the ***natural domain*** of the function.

▶ **Example 6** Find the natural domain of

(a) $f(x) = x^3$ (b) $f(x) = 1/[(x-1)(x-3)]$

Solution (a)

Solution (b)

c) $f(x) = \tan x$

d) $f(x) = \sqrt{x^2 - 5x + 6}$

▶ Example 7 The natural domain of the function

$$f(x) = \frac{x^2 - 4}{x - 2} \tag{2}$$

▸ **Example 8** Find the domain and range of

(a) $f(x) = 2 + \sqrt{x - 1}$ (b) $f(x) = (x + 1)/(x - 1)$

Solution (a)

Solution (b)

Homework:

1, 3, 7, 9 (a-c), 10 (a-c-e), 15, 18 pages (11- 13)