



Exercise set 0.4 :

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1, 3(a, b, c, d, f) 9, 10, 11, 12, 13, 17, 18, 19, 22 p.44-45.

1. In (a)–(d), determine whether f and g are inverse functions.

(a) $f(x) = 4x$, $g(x) = \frac{1}{4}x$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{4}x\right) = 4\left(\frac{1}{4}x\right) = x$$

$$(g \circ f)(x) = g(f(x)) = g(4x) = \frac{1}{4}(4x) = x$$

$\therefore f$ and g are inverse function

(b) $f(x) = 3x + 1$, $g(x) = 3x - 1$

$$(f \circ g)(x) = f(g(x)) = f(3x - 1)$$

$$= 3(3x - 1) + 1 = 9x - 3 + 1$$

$$= 9x - 2$$

$\therefore f$ and g are not inverse function

$$(c) \quad f(x) = \sqrt[3]{x-2}, \quad g(x) = x^3 + 2$$

$$(f \circ g)(x) = f(g(x)) = f(x^3 + 2) = \sqrt[3]{x^3 + 2} - 2$$

$$= \sqrt[3]{x^3} = x$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x-2}) = (\sqrt[3]{x-2})^3 + 2$$

$$= x - 2 + 2$$

$$= x$$

$\therefore f$ and g are inverse function

$$(d) \quad f(x) = x^4, \quad g(x) = \sqrt[4]{x}$$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt[4]{x}) = (\sqrt[4]{x})^4 = x$$

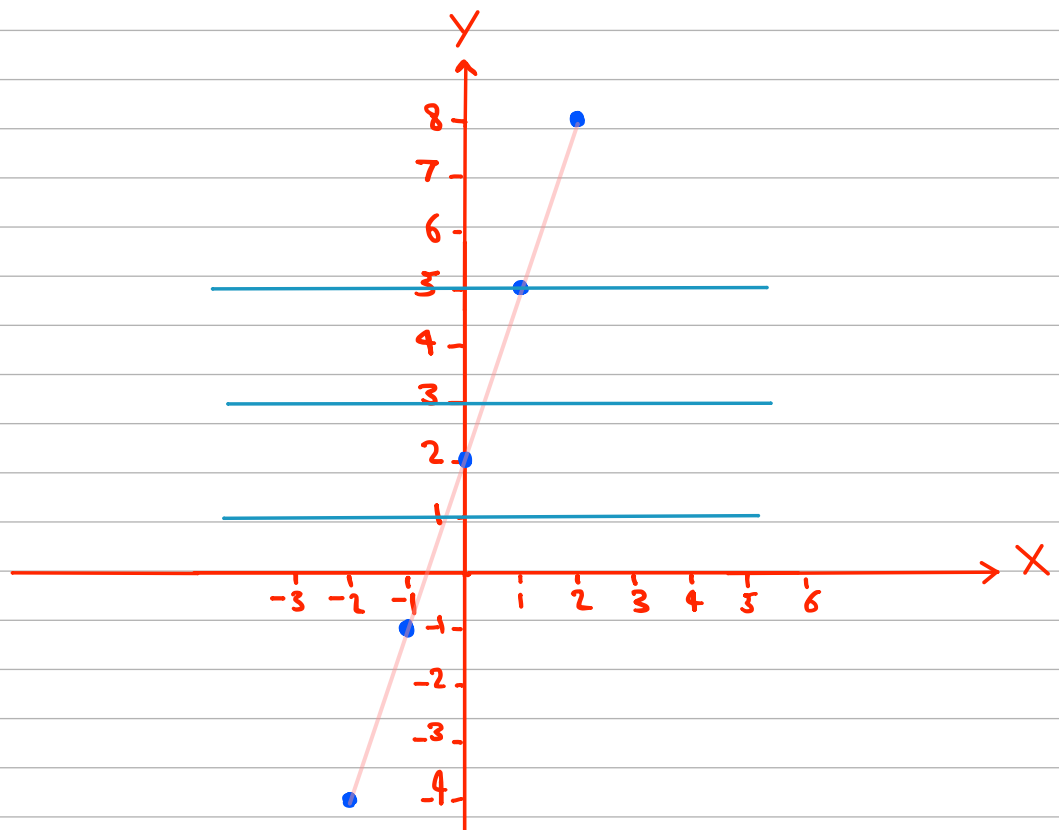
$$(g \circ f)(x) = g(f(x)) = g(x^4) = \sqrt[4]{x^4} = x$$

$\therefore f$ and g are inverse function

3. In each part, use the horizontal line test to determine whether the function f is one-to-one.

(a) $f(x) = 3x + 2$

x	$y = 3x + 2$	(x, y)
-2	$3(-2) + 2 = -4$	$(-2, -4)$
-1	$3(-1) + 2 = -1$	$(-1, -1)$
0	$3(0) + 2 = 2$	$(0, 2)$
1	$3(1) + 2 = 5$	$(1, 5)$
2	$3(2) + 2 = 8$	$(2, 8)$



one to one function

$$(b) f(x) = \sqrt{x-1}$$

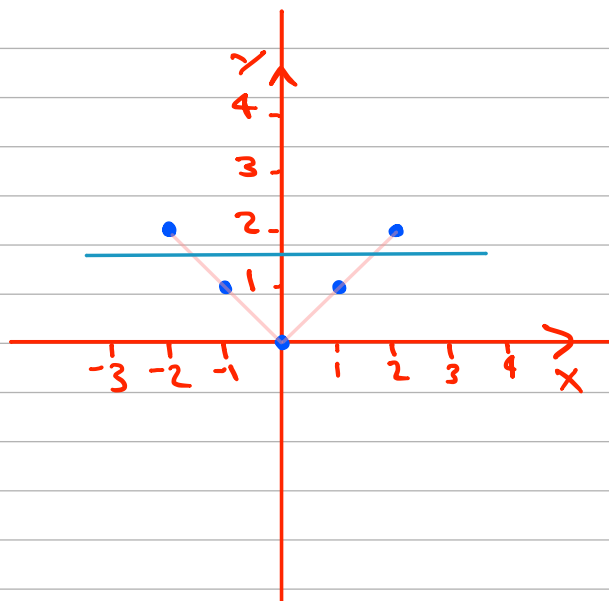
X	$Y = \sqrt{X-1}$	(X, Y)
1	$\sqrt{1-1} = 0$	$(1, 0)$
2	$\sqrt{2-1} = \sqrt{1} = 1$	$(2, 1)$
3	$\sqrt{3-1} = \sqrt{2} = 1.4$	$(3, 1.4)$
4	$\sqrt{4-1} = \sqrt{3} = 1.7$	$(4, 1.7)$



one to one function

(c) $f(x) = |x|$

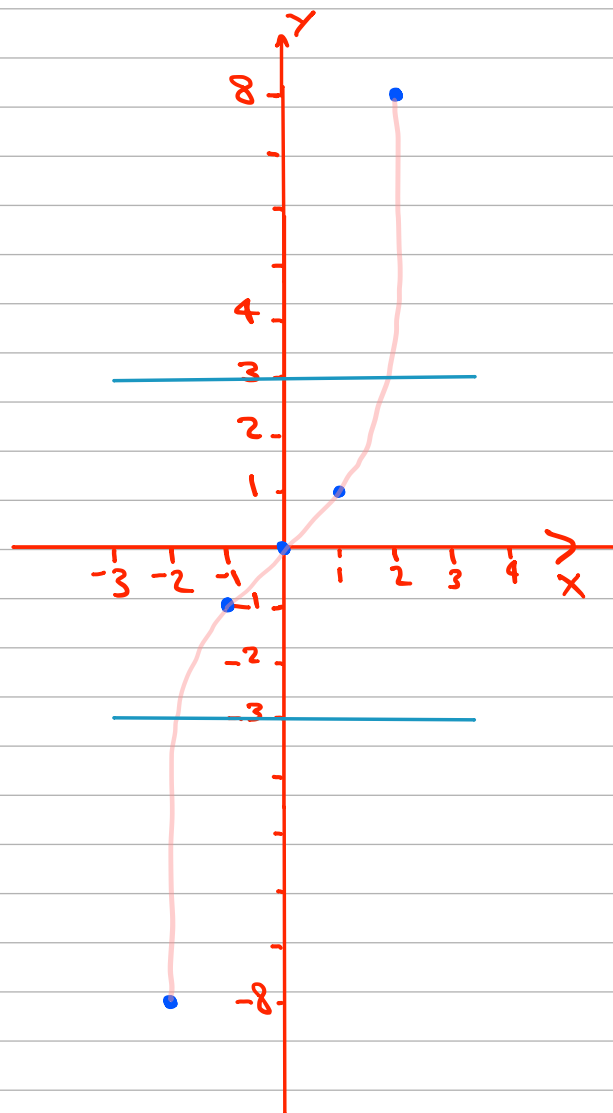
x	$y=f(x)= x $	(x,y)
-2	$ -2 =2$	$(-2,2)$
-1	$ -1 =1$	$(-1,1)$
0	$ 0 =0$	$(0,0)$
1	$ 1 =1$	$(1,1)$
2	$ 2 =2$	$(2,2)$



not one to one function

(d) $f(x) = x^3$

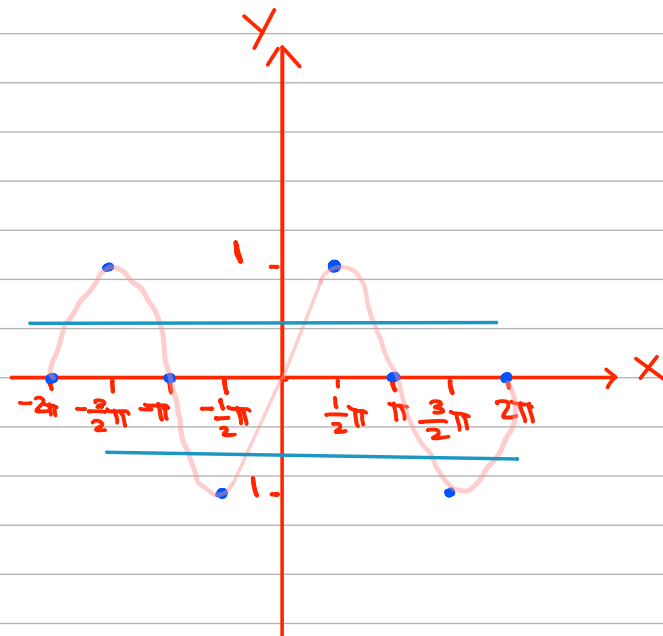
x	$y = f(x) = x^3$	(x, y)
-2	$(-2)^3 = -8$	$(-2, -8)$
-1	$(-1)^3 = -1$	$(-1, -1)$
0	$(0)^3 = 0$	$(0, 0)$
1	$(1)^3 = 1$	$(1, 1)$
2	$(2)^3 = 8$	$(2, 8)$



one to one function

$$(f) \quad f(x) = \sin x$$

x	$y = \sin x$	(x, y)
$\frac{1}{2}\pi$	$\sin(\frac{1}{2}\pi) = 1$	$(\frac{1}{2}\pi, 1)$
π	$\sin(\pi) = 0$	$(\pi, 0)$
2π	$\sin(2\pi) = 0$	$(2\pi, 0)$
$\frac{3}{2}\pi$	$\sin(\frac{3}{2}\pi) = -1$	$(\frac{3}{2}\pi, -1)$
$-\frac{3}{2}\pi$	$= 1$	$(-\frac{3}{2}\pi, 1)$
-2π	$= 0$	$(-2\pi, 0)$
$-\pi$	$= 0$	$(-\pi, 0)$
$-\frac{1}{2}\pi$	$= -1$	$(-\frac{1}{2}\pi, -1)$



one to one function

9–16 Find a formula for $f^{-1}(x)$. ■

9. $f(x) = 7x - 6$

$$Y = 7x - 6$$

$$Y + 6 = 7x$$

$$x = \frac{Y + 6}{7}$$

$$f^{-1}(y) = \frac{Y + 6}{7}$$

$$f^{-1}(x) = \frac{x + 6}{7}$$

10. $f(x) = \frac{x+1}{x-1}$

$$y = \frac{x+1}{x-1}$$

$$y(x-1) = (x+1)$$

$$yx - y = x + 1$$

$$yx - x = y + 1$$

$$x(y-1) = y+1$$

$$x = \frac{y+1}{y-1}$$

$$f^{-1}(y) = \frac{y+1}{y-1}$$

$$f^{-1}(x) = \frac{x+1}{x-1}$$

$$11. f(x) = 3x^3 - 5$$

$$Y = 3X^3 - 5$$

$$Y + 5 = 3X^3$$

$$X^3 = \frac{Y + 5}{3}$$

$$\sqrt[3]{X^3} = \sqrt[3]{\frac{Y + 5}{3}}$$

$$X = \sqrt[3]{\frac{Y + 5}{3}}$$

$$f^{-1}(Y) = \sqrt[3]{\frac{Y + 5}{3}}$$

$$f^{-1}(X) = \sqrt[3]{\frac{X + 5}{3}}$$

12. $f(x) = \sqrt[5]{4x+2}$

$$y = \sqrt[5]{4x+2}$$

$$(y)^5 = (\sqrt[5]{4x+2})^5$$

$$y^5 = 4x+2$$

$$y^5 - 2 = 4x$$

$$x = \frac{y^5 - 2}{4}$$

$$f^{-1}(y) = \frac{y^5 - 2}{4}$$

$$f^{-1}(x) = \frac{x^5 - 2}{4}$$

13. $f(x) = 3/x^2, \quad x < 0$

$$y = \frac{3}{x^2}$$

$$y x^2 = 3$$

$$x^2 = \frac{3}{y}$$

$$\sqrt{x^2} = \sqrt{\frac{3}{y}}$$

$$x = \sqrt{\frac{3}{y}}$$

$$f^{-1}(y) = \sqrt{\frac{3}{y}}$$

$$f^{-1}(x) = -\sqrt{\frac{3}{x}}$$

17–20 Find a formula for $f^{-1}(x)$, and state the domain of the function f^{-1} . ■

17. $f(x) = (x + 2)^4, \quad x \geq 0$

$$Y = (X + 2)^4$$

$$\sqrt[4]{Y} = \sqrt[4]{(X + 2)^4}$$

$$\sqrt[4]{Y} = X + 2$$

$$X = \sqrt[4]{Y} - 2$$

$$f^{-1}(x) = \sqrt[4]{x} - 2$$

Domain:

$D_{f^{-1}} = [0, +\infty)$ because the condition $x \geq 0$

18. $f(x) = \sqrt{x+3}$

$$Y = \sqrt{X+3}$$

$$(Y)^2 = (\sqrt{X+3})^2$$

$$Y^2 = X+3$$

$$X = Y^2 - 3$$

$$f^{-1}(X) = X^2 - 3$$

Domain:

$$D_f = X+3 \geq 0 = X \geq -3 = [-3, +\infty)$$

Range:

$$X \geq -3 \rightarrow X+3 \geq -3+3 \rightarrow X+3 \geq 0$$

$$\sqrt{X+3} \geq \sqrt{0} \rightarrow \sqrt{X+3} \geq 0 \rightarrow [0, +\infty)$$

$$R_f = D_{f^{-1}} = [0, +\infty)$$

$$D_{f^{-1}} = [0, +\infty)$$

$$19. f(x) = -\sqrt{3-2x}$$

$$y = -\sqrt{3-2x}$$

$$y^2 = (-\sqrt{3-2x})^2$$

$$y^2 = 3-2x$$

$$y^2 - 3 = -2x$$

$$x = \frac{y^2 - 3}{-2}$$

$$f^{-1}(x) = \frac{x^2 - 3}{-2}$$

Domain:

$$D_f = 3-2x \geq 0 \rightarrow 3 \geq 2x \rightarrow x \leq \frac{3}{2} \rightarrow (-\infty, \frac{3}{2}]$$

Range:

$$x \leq \frac{3}{2} \rightarrow 2x \leq 3 \rightarrow 2x-3 \leq 3-3$$

$$\sqrt{2x-3} \leq \sqrt{0} \rightarrow \sqrt{2x-3} \leq 0 \rightarrow (-\infty, 0]$$

$$R_f = D_{f^{-1}} = (-\infty, 0]$$

$$D_{f^{-1}} = (-\infty, 0]$$

21–24 True–False Determine whether the statement is true or false. Explain your answer. ■

22. If f and g are inverse functions, then f and g have the same domain. (F)

$$f(x) = 1 + \frac{1}{x} \rightarrow D_f = \mathbb{R} \setminus \{0\}$$

$$g(x) = \frac{1}{x-1} \rightarrow D_g = \mathbb{R} \setminus \{1\}$$