

**0.2 NEW FUNCTIONS FROM OLD****ARITHMETIC OPERATIONS ON FUNCTIONS**

**0.2.1 DEFINITION** Given functions  $f$  and  $g$ , we define

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$(f/g)(x) = f(x)/g(x)$$

For the functions  $f + g$ ,  $f - g$ , and  $fg$  we define the domain to be the intersection of the domains of  $f$  and  $g$ , and for the function  $f/g$  we define the domain to be the intersection of the domains of  $f$  and  $g$  but with the points where  $g(x) = 0$  excluded (to avoid division by zero).



► **Example 1** Let

$$f(x) = 1 + \sqrt{x - 2} \quad \text{and} \quad g(x) = x - 3$$

Find the domains and formulas for the functions  $f + g$ ,  $f - g$ ,  $fg$ ,  $f/g$ , and  $7f$ .

***Solution.***



## COMPOSITION OF FUNCTIONS



**0.2.2 DEFINITION** Given functions  $f$  and  $g$ , the *composition* of  $f$  with  $g$ , denoted by  $f \circ g$ , is the function defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of  $f \circ g$  is defined to consist of all  $x$  in the domain of  $g$  for which  $g(x)$  is in the domain of  $f$ .

► **Example 3** Let  $f(x) = x^2 + 3$  and  $g(x) = \sqrt{x}$ . Find

(a)  $(f \circ g)(x)$       (b)  $(g \circ f)(x)$

***Solution (a)***

**Solution (b)**

► **Example 4** Find  $(f \circ g \circ h)(x)$  if

$$f(x) = \sqrt{x}, \quad g(x) = 1/x, \quad h(x) = x^3$$

**EXPRESSING A FUNCTION AS A COMPOSITION**

► **Example 5** Express  $\sin(x^3)$  as a composition of two functions.

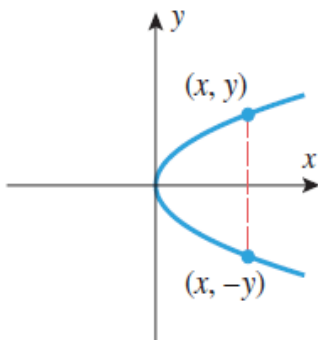
*Solution*



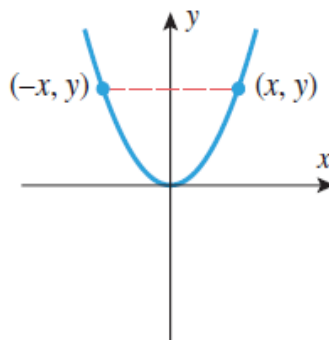
<i>FUNCTION</i>	<i><math>g(x)</math> INSIDE</i>	<i><math>f(x)</math> OUTSIDE</i>	<i>COMPOSITION</i>

**REMARK**

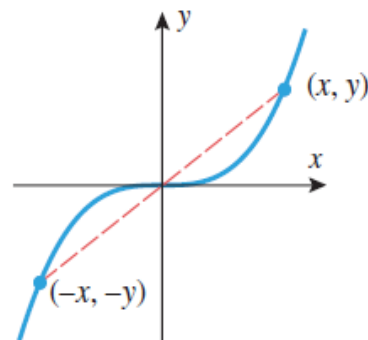
## SYMMETRY



Symmetric about  
the  $x$ -axis



Symmetric about  
the  $y$ -axis



Symmetric about  
the origin



### 0.2.3 THEOREM (Symmetry Tests)

- A plane curve is symmetric about the  $y$ -axis if and only if replacing  $x$  by  $-x$  in its equation produces an equivalent equation.
- A plane curve is symmetric about the  $x$ -axis if and only if replacing  $y$  by  $-y$  in its equation produces an equivalent equation.
- A plane curve is symmetric about the origin if and only if replacing both  $x$  by  $-x$  and  $y$  by  $-y$  in its equation produces an equivalent equation.

## EVEN AND ODD FUNCTIONS

A function  $f$  is said to be an *even function* if

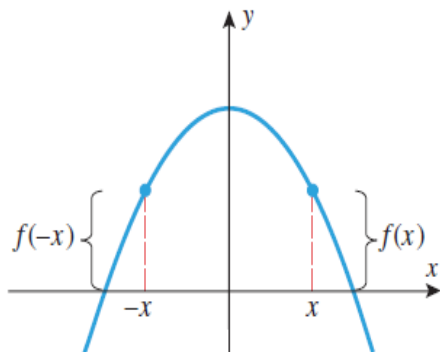
$$f(-x) = f(x) \quad (8)$$

and is said to be an *odd function* if

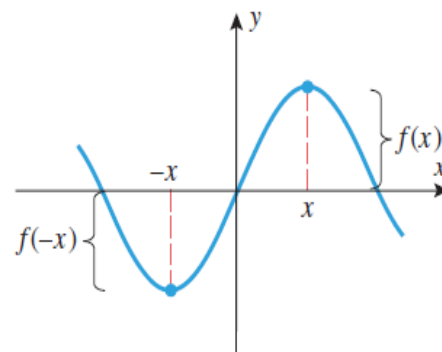
$$f(-x) = -f(x) \quad (9)$$

Geometrically, the graphs of even functions are symmetric about the  $y$ -axis because replacing  $x$  by  $-x$  in the equation  $y = f(x)$  yields  $y = f(-x)$ , which is equivalent to the original equation  $y = f(x)$  by (8) (see Figure 0.2.9).

Similarly, it follows from (9) that graphs of odd functions are symmetric about the origin (see Figure 0.2.10).



▲ Figure 0.2.9 This is the graph of an even function since  $f(-x) = f(x)$ .



▲ Figure 0.2.10 This is the graph of an odd function since  $f(-x) = -f(x)$ .

Some examples of even functions:

Some examples of odd functions:

**► Example**

**Exercise 59 Page 26:** In each part, classify the function as even, odd, or neither

(c)  $f(x) = |x|$

(e)  $f(x) = \frac{x^5 - x}{1 + x^2}$

**Homework:**

27, 31, 35, 59 (b-d-f) pages (25- 26)

## \*) graphs of functions :

$f(x) + c \longrightarrow$  Shifting up by  $(c)$

$f(x) - c \longrightarrow$  Shifting down by  $(c)$

$f(x+c) \longrightarrow$  Shifting left by  $(c)$

$f(x-c) \longrightarrow$  Shifting right by  $(c)$

$-f(x) \longrightarrow$  Reflection about x-axis

$f(-x) \longrightarrow$  Reflection about y-axis

$a f(x) \longrightarrow$  Vertical stretching

$\frac{1}{a} f(x) \longrightarrow$  Vertical compressing

$f(ax) \longrightarrow$  Horizontal compressing

$f(\frac{1}{a}x) \longrightarrow$  Horizontal stretching

