حساب التفاضل والتكامل 1- ريض 101ت

Calculus -Masc 101T

Text Book:

H. Anton, I. Bivens, and S. Davis,

Calculus: Late Transcendental Single and multivariable, 9th Edition

Chapters

0,1,2,3,4,6

0 **BEFORE CALCULUS** 1

الدوال

0.1 Functions

متغير يعتمد

> **0.1.1 DEFINITION** If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that ***y is a function of x***.

ايجاد بالضبط دالة

تمثيل

Four common methods for representing functions are:

طرق

- **Numerically by tables**

A	2
B	3
C	4
D	5
E	6

function

A	2
B	3
A	4
D	5
E	6

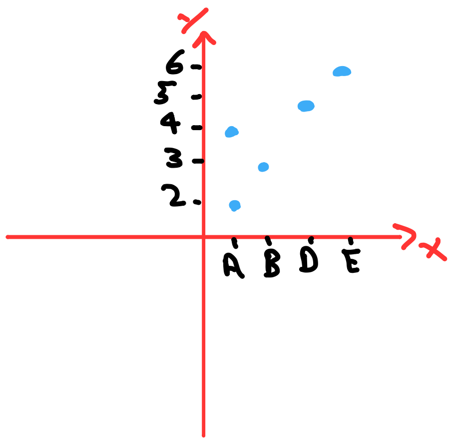
not function

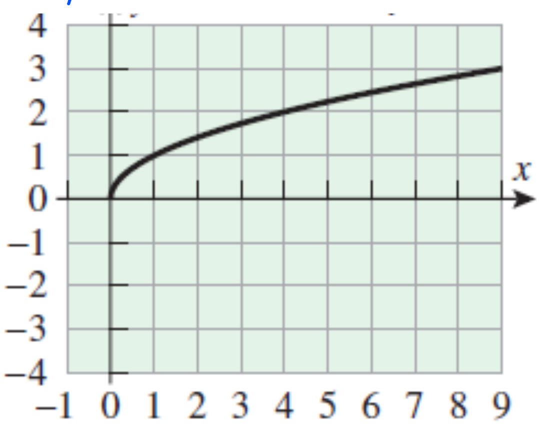
A	2
B	3
C	2
D	5
E	6

function

- **Geometrically by graphs**

الرسم





- **Algebraically by formulas**

- **Verbally**

تشارك وحيد مخرج لكل

> **0.1.2 DEFINITION** A *function* f is a rule that associates a unique output with each input. If the input is denoted by x, then the output is denoted by $f(x)$ (read "f of x").

مدخل يرمز

مستقل تابع متغير

INDEPENDENT AND DEPENDENT VARIABLES

قيمة

For a given input x, the output of a function f is called the ***value*** of f at x or the ***image*** of x under f.

Examples صورة تحت

$f(x) = 3x + 2$

$x \to$ independent

$f(x) \to$ dependent

نرمز حرف مفرد

Sometimes we will want to denote the output by a single letter, say y, and write

$$y = f(x)$$

معادلة تعبر

- This equation expresses y as a function of x;

تسمى مستقل متغير

- The variable x is called the ***independent variable*** (or ***argument***) of f

متغير تابع

- The variable y is called the ***dependent variable*** of f.

- If the independent and dependent variables are real numbers, in which case we say that f is a ***real-valued function of a real variable***.

Table 0.1.2

x	0	1	2	3
y	3	4	−1	6

العلاقة

Example 1 Table 0.1.2 describes a functional relationship $y = f(x)$ for which

$f(0) = 3$ — f associates $y = 3$ with $x = 0$.

$f(1) = 4$ — f associates $y = 4$ with $x = 1$.

$f(2) = -1$ — f associates $y = -1$ with $x = 2$.

$f(3) = 6$ — f associates $y = 6$ with $x = 3$.

Example 2 The equation

$$y = 3x^2 - 4x + 2$$

معادلة

has the form $y = f(x)$ in which the function f is given by the formula

$$f(x) = 3x^2 - 4x + 2$$

$$f(0) = 3(0)^2 - 4(0) + 2 = 2$$

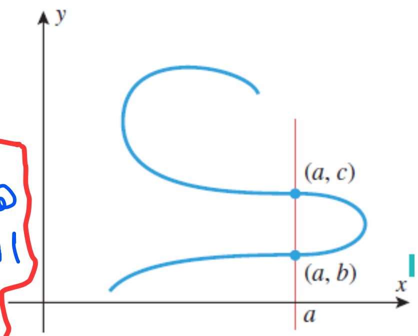
$$f(2) = 3(2)^2 - 4(2) + 2 =$$

اختبار الخط العمودي

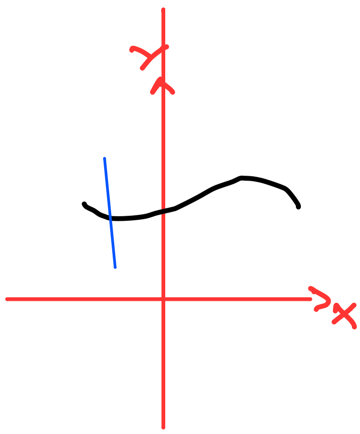
0.1.3 THE VERTICAL LINE TEST *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*

اذا قطع الخط العمودي الدالة في اكثر من نقطة اذا هذه ليست دالة

هذه ليست دالة لأن الخط العمودي قطع الدالة في اكثر من نقطة



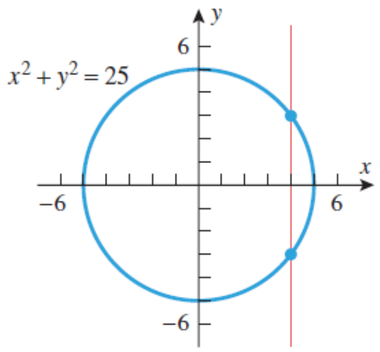
▲ **Figure 0.1.7** This curve cannot be the graph of a function.



هذه دالة لأن الخط العمودي قطع في نقطة واحد

► **Example 3** The graph of the equation

$$x^2 + y^2 = 25$$



▲ **Figure 0.1.8**

القيمة المطلقة

THE ABSOLUTE VALUE FUNCTION

Recall that the ***absolute value*** or ***magnitude*** of a real number x is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative. Thus,

$$|5| = 5, \quad \left|-\tfrac{4}{7}\right| = \tfrac{4}{7}, \quad |0| = 0$$

مميزات اعداد حقيقة

0.1.4 PROPERTIES OF ABSOLUTE VALUE *If a and b are real numbers, then*

سالب نفس

(a) $|-a| = |a|$ A number and its negative have the same absolute value.

ضرب

(b) $|ab| = |a|\,|b|$ The absolute value of a product is the product of the absolute values.

نسبة

(c) $|a/b| = |a|/|b|, b \neq 0$ The absolute value of a ratio is the ratio of the absolute values.

المتباينة المثلثية

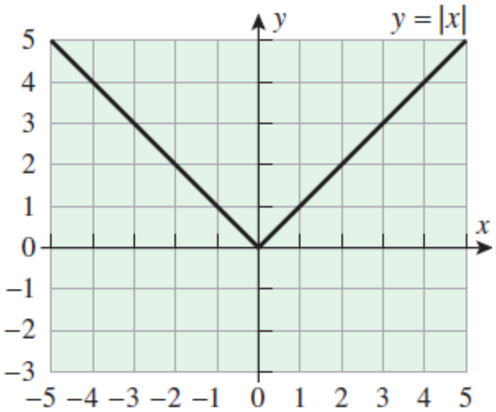
(d) $|a + b| \leq |a| + |b|$ The *triangle inequality*

رسم

The graph of the function $f(x) = |x|$ can be obtained by graphing the two parts of the equation

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

separately. Combining the two parts produces the V-shaped graph in Figure 0.1.9.



▲ **Figure 0.1.9**

Remark:

$\sqrt{x^2} = x$ This equation correct if x is nonnegative

- if x negative if $x = -4$ then $\sqrt{x^2} = \sqrt{(-4)^2} = \sqrt{16}$
$= 4 \neq x$

$\sqrt{x^2} = |x|$

التعريف المختلفة الدالة

PIECEWISE-DEFINED FUNCTIONS

The absolute value function $f(x) = |x|$ is an example of a function that is defined ***piecewise*** in the sense that the formula for f changes, depending on the value of x.

▶ Example

8. Find $g(3)$, $g(-1)$, $g(\pi)$, $g(-1.1)$, and $g(t^2-1)$.

$$\text{(b)}\quad g(x) = \begin{cases} \sqrt{x+1}, & x \geq 1 \\ 3, & x < 1 \end{cases}$$

Solution.

$g(3) = \sqrt{x+1} = \sqrt{3+1} = \sqrt{4} = 2$

$g(-1) = 3$

$g(\pi) = \sqrt{\pi + 1} = \sqrt{3.14+1} = \sqrt{4.14}$

$g(-1.1) = 3$

$$g(t^2-1) = \begin{cases} \sqrt{t^2-1+1} = \sqrt{t^2} = |t| & t^2-1 \geq 1 \\ 3 & t^2-1 < 1 \end{cases}$$

$t^2-1 \geq 1$

$t^2 \geq 2$

$\sqrt{t^2} \geq \sqrt{2}$

$|t| \geq \sqrt{2}$

$|a| \geq b \Rightarrow a \geq b$ or $a \leq -b$

$t \geq \sqrt{2}$, $t \leq -\sqrt{2}$

$t^2-1 < 1$

$t^2 < 2$

$\sqrt{t^2} < \sqrt{2}$

$|t| < \sqrt{2}$

$|a| < b \Rightarrow -b < a \leq b$

$-\sqrt{2} < t < \sqrt{2}$

$$g(t^2-1) = \begin{cases} t & t \geq \sqrt{2} \\ -t & t \leq -\sqrt{2} \\ 3 & -\sqrt{2} < t < \sqrt{2} \end{cases}$$

المجال المدى

DOMAIN AND RANGE

المسموحة

مجموعة المخرجات

If x and y are related by the equation $y = f(x)$, then the set of all allowable inputs (x-values) is called the ***domain*** of f, and the set of outputs (y-values) that result when x varies over the domain is called the ***range*** of f.

> **0.1.5 DEFINITION** If a real-valued function of a real variable is defined by a formula, and if no domain is stated explicitly, then it is to be understood that the domain consists of all real numbers for which the formula yields a real value. This is called the ***natural domain*** of the function.

▶ **Example 6** Find the natural domain of

(a) $f(x) = x^3$ (b) $f(x) = 1/[(x-1)(x-3)]$

Solution (a)

$f(x) = x^3$

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the natural domain is $(-\infty, \infty)$

Solution (b)

$$f(x) = \frac{1}{(x-1)(x-3)}$$

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$x - 1 = 0 \Rightarrow x = 1$

$x - 3 = 0 \Rightarrow x = 3$

domain all real number except $x = 1, 3$

$\{x : x \neq 1 \text{ and } x \neq 3\} = (-\infty, 1) \cup (1, 3) \cup (3, \infty)$

c) $f(x) = \tan x$

$\tan x = \dfrac{\sin x}{\cos x}$

domain all real number except when $\cos x = 0$

$\cos x = 0$ when x is odd integer multiple of $\dfrac{\pi}{2}$

d) $f(x) = \sqrt{x^2 - 5x + 6}$

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الجذر اكبرمن او يساوي صفر

$f(x) = \sqrt{(x-2)(x-3)}$

$(x-2)(x-3) \geq 0$

$x = 2 , x = 3$

$x < 2$

$(1-2)(1-3) = -1 \cdot -2 = 2 \rightsquigarrow x \geq 0$

$2 < x < 3$

$(2.5-2)(2.5-3) = \frac{1}{2} \cdot -\frac{1}{2} = -\frac{1}{4} \rightsquigarrow x < 0$

$x > 3$

$(4-2)(4-3) = 2 \cdot 1 = 2 \rightsquigarrow x \geq 0$

$D = (-\infty, 2] \cup [3, \infty)$

Example 7 The natural domain of the function

$$f(x) = \frac{x^2 - 4}{x - 2} \tag{2}$$

$x - 2 \neq 0$

$x = 2$

$D = R \setminus \{2\} = (-\infty, 2) \cup (2, \infty)$

▶ **Example 8** Find the domain and range of

(a) $f(x) = 2 + \sqrt{x-1}$ (b) $f(x) = (x+1)/(x-1)$

Solution (a)

$f(x) = 2 + \sqrt{x-1}$

Domain:

$x - 1 \geq 1 \Rightarrow x \geq 1 \Rightarrow [1, +\infty)$

Range:

$f(1) = 2 + \sqrt{1-1} = 2$

$f(2) = 2 + \sqrt{2-1} = 3$

$[2, +\infty)$

$x - 1 \geq 0$

$\sqrt{x-1} \geq \sqrt{0}$

$2 + \sqrt{x-1} \geq 2$

Solution (b)

$$f(x) = \frac{x+1}{x-1}$$

domain:

$$x-1=0 \Longrightarrow x=1$$

$$\{x : x \neq 1\} = (-\infty, 1) \cup (1, +\infty)$$

Range:

$$f(x) = \frac{x+1}{x-1}$$

$$y = \frac{x+1}{x-1}$$

$$y(x-1) = x+1$$

$$yx - y = x+1$$

$$yx - x = 1+y$$

$$x(y-1) = 1+y$$

$$x = \frac{y+1}{y-1}$$

$$f^{-1}(x) = \frac{y+1}{y-1}$$

$$D_{f^{-1}(x)} = R_f$$

$$D_{f^{-1}(x)} = x-1 \neq 0 \Rightarrow x \neq 1 = (-\infty, 1) \cup (1, +\infty)$$

$$R_f = \mathbb{R} \setminus \{1\} = (-\infty, 1) \cup (1, +\infty)$$

Homework:

1, 3, 7, 9 (a-c), 10 (a-c-e), 15, 18 pages (11- 13)